

Trigonometric Functions and Graphs

$$y = a + b\cos cx \quad \text{or} \quad y = a + b\sin cx$$

$$\text{Period} = \frac{360}{c} \text{ degrees} \quad \text{Range} = [a-b, a+b]$$

Example (given a function)

$$y = 4 - 2\cos 3x$$

$$\text{Period} = \frac{360}{3} = 120 \text{ degrees}$$

$$\text{Range} = [2, 6]$$

$$\text{based on } 4 - 2 = 2 \text{ and } 4 + 2 = 6$$

Example

$$y = 3\sin 6x$$

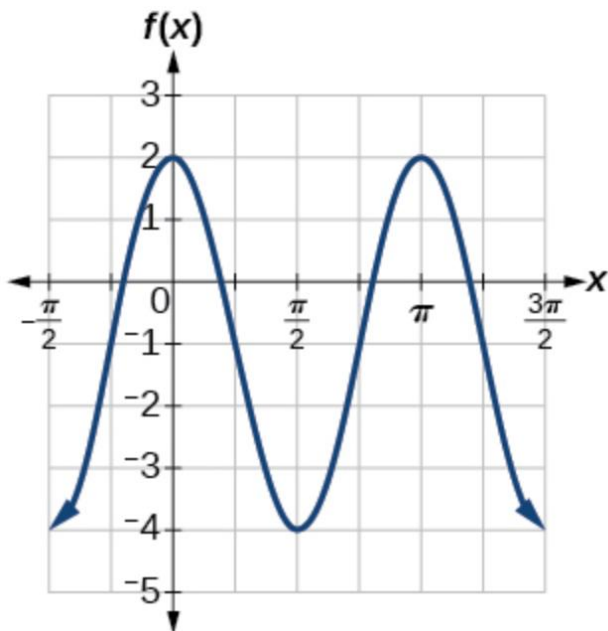
$$\text{Period} = \frac{360}{6} = 60 \text{ degrees}$$

$$\text{Range} = [-3, 3]$$

$$\text{based on } 0 - 3 = -3 \text{ and } 0 + 3 = 3$$

Example (given a graph)

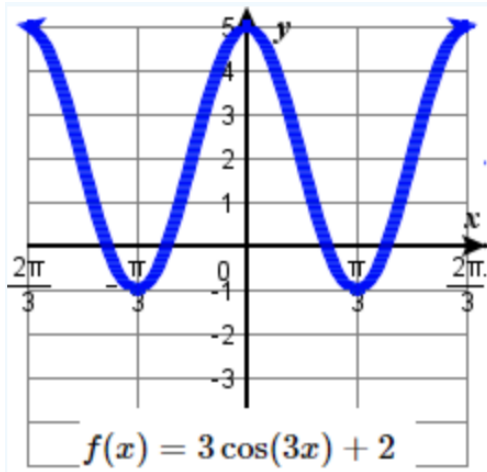
What is the period and range of the following function $f(x)$?



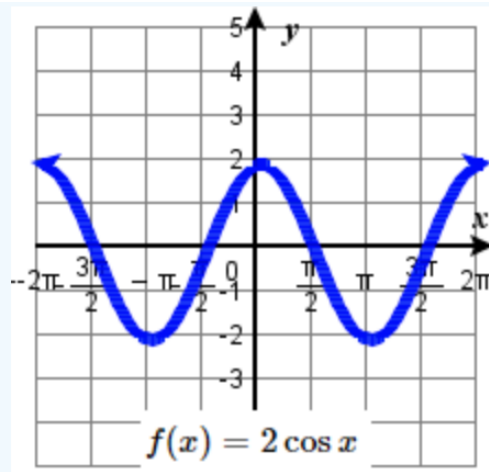
$$\text{Period} = 180 \text{ degrees}$$

$$\text{Range} = [-4, 2]$$

Example

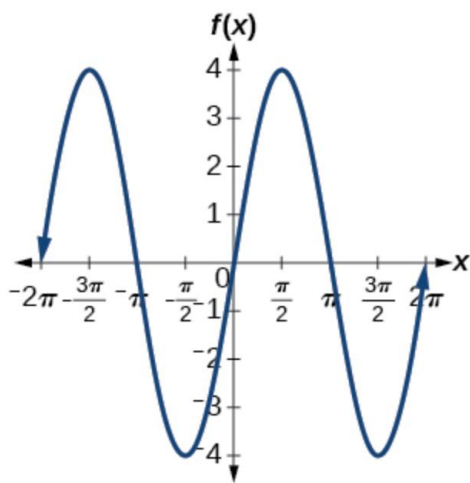


Period = $\frac{360}{3} = 120$ Range = $[-1, 5]$



Period = $\frac{360}{1} = 360$ Range = $[-2, 2]$

Example

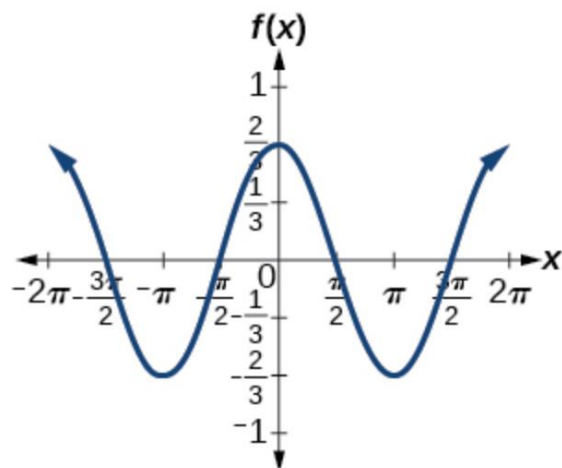


What is the period and range of the graph above?

Period = 360 degrees Range = $[-4, 4]$

Function: $f(x) = 4 \sin x$

Example

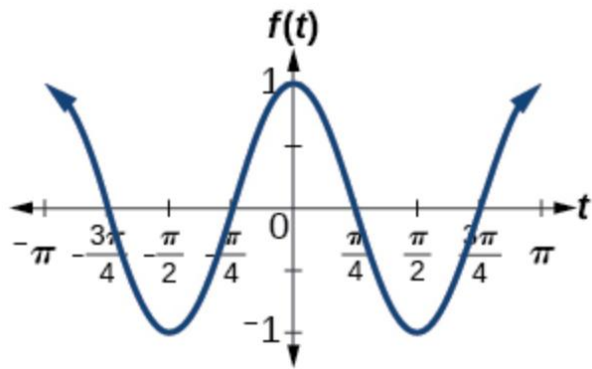


What is the period and range of the graph above?

Period = 360 degrees Range = $[-\frac{2}{3}, \frac{2}{3}]$

Function: $f(x) = \frac{2}{3} \cos x$

Example



What is the period and range of the graph above?

$$\text{Period} = \frac{360}{2} = 180 \text{ degrees}$$

$$\text{Range} = [-1, 1]$$

$$\text{Function: } f(x) = \cos 2x$$

Real-World Applications

Example

A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meter above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function $h(t)$ gives a person's height in meters above the ground t minutes after the wheel begins to turn

- Find the amplitude, midline, and period of $h(t)$?
- Find a formula for the height?
- How high off the ground is a person after 55 minutes?

Answer

- Amplitude: 12.5; period: 10; midline: $y = 13.5$
- $h(t) = 12.5 \sin\left(\frac{\pi}{5}(t - 2.5)\right) + 13.5$
- 26 ft

Resource Reference:

https://math.libretexts.org/Courses/Monroe_Community_College/MTH_165_College_Algebra_MTH_175_Precalculus/05%3A_Trigonometric_Functions_and_Graphs/5.5%3A_Graphs_of_the_Sine_and_Cosine_Functions/5.5e%3A_Exercises_-_Graphs_of_Sine_and_Cosine_Functions