Cork College of FET
Cork's Further Education & Training Service
TRAMORE ROAD CAMPUS
A Pathway for Every Learner

Component:	Mathematics 5N1833 – Trial Paper Solutions
Month:	2024

Course(s):

AA [Applied Science: Laboratory Techniques]

AB [Applied Biology: Food Health & Nutrition]

Extra Maths

Total Marks: 800marks

Weighting: 40%

Time Allowed: 2 Hours

Section A (400 Marks)
10 short questions.
Answer **ALL** 10
40 marks each

1. Determine the equation of a line in the form $y = mx + c$ if the points (1,2) and (3,4) are on the line **40 marks**

Slope m = $\frac{y2-y1}{x2-x1} = \frac{4-2}{3-1} = 1$

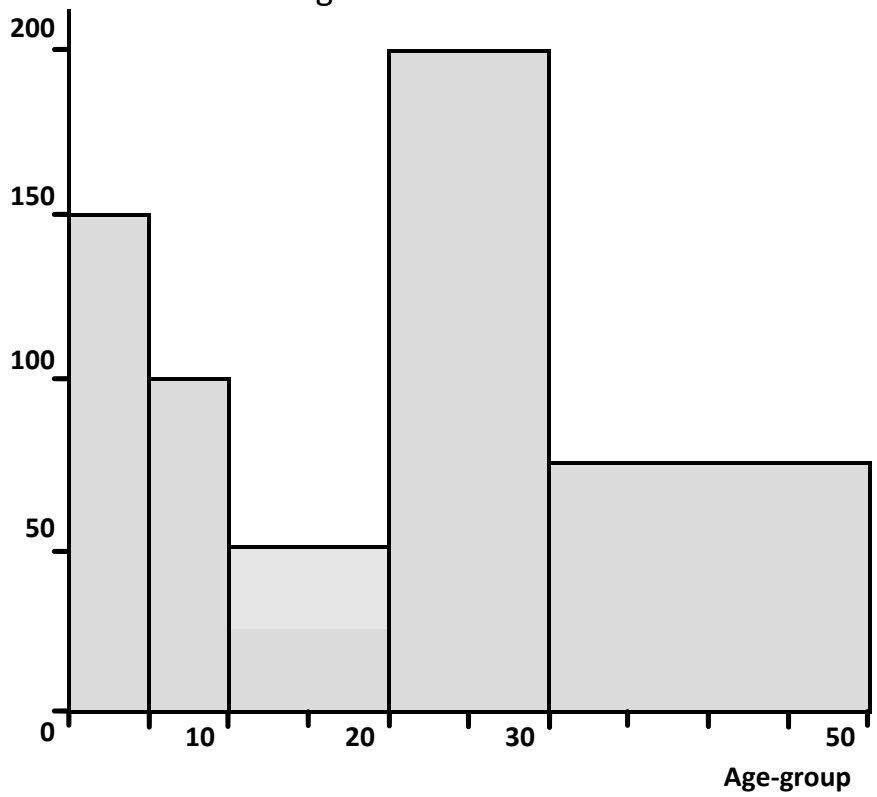
Equation = $y - y1 = m(x - x1)$

$y - 2 = 1(x - 1)$

$y - 2 = x - 1$

$y = x + 1$

2. The age distribution of a group of people who wear glasses is shown on this histogram.



If there are 200 people in the 20 – 30 age-group, find

(i) The number of people in the 30 – 50 age group 150 **20 marks**

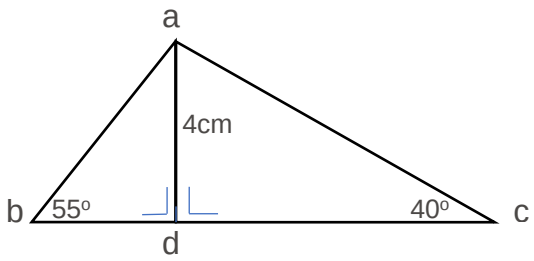
(ii) The total number of people wearing glasses = **20 marks**
75+50+50+200+150 = 525

3. How many different 3-digit numbers can be formed from the digits 1, 2, 3, 4

(i) If no digit is repeated in the number? $= 4 * 3 * 2 = 24$ **20 marks**

(ii) How many of these begin with 3? = $1 * 3 * 2 = 6$ **20 marks**

4.



In the given triangle abc, ad ⊥ bc.
If |ad| = 4cm, |∠abd| = 55° and |∠acd| = 40°,

(i) Find |bc| to 1 decimal place **20 marks**

$$tan40 = \frac{4}{dc} \qquad dc = \frac{4}{tan40} = 4.8$$
$$tan55 = \frac{4}{bd} \qquad bd = \frac{4}{tan55} = 2.8$$
$$bc = bd + dc$$
$$= 4.8 + 2.8 = 7.6\text{cm}$$

(ii) Find |ac| to 1 decimal place **20 marks**

$$ac^2 = ad^2 + dc^2$$
$$ac^2 = 4^2 + 4.8^2$$
$$= 39.04$$
$$\therefore ac = \sqrt{39.04} = 6.2$$

5. Using differentiation, calculate the slope of the tangent to the curve **40 marks**

$$y = 2x^3 - x^2 + 5 \quad \text{at} \quad x = 3$$
$$\frac{dy}{dx} = 6x^2 - 2x$$
$$\text{Slope } when\ x = 3, = \frac{dy}{dx} = 6(3^2) - 2(3) = 48$$

6.

$$\text{If } f(x) = \frac{x-1}{4}$$

(i) Find $f(6)$ **20 marks**

$$f(6) = \frac{6-1}{4} = \frac{5}{4}$$

(ii) Find $f^{-1}(3)$ **20 marks**

$$x \to \frac{x-1}{4}$$
$$4x \to x - 1$$
$$4x + 1 \to x$$
$$f^{-1}(x) = 4x + 1$$
$$f^{-1}(3) = 4(3) + 1 = 13$$

7. $y = 2sinx + 2e^{4x}$ find the derivative $\frac{dy}{dx}$ **40 marks**

$$\frac{dy}{dx} = 2cosx + 8e^{4x}$$

8. Evaluate $\int(3x^3 - cos2x + e^{4x})dx$ **40 marks**

$$= \frac{3x^4}{4} - \frac{\sin 2x}{2} + \frac{e^{4x}}{4} + c$$

9. If $w = 3 - 5i$ and $z = 4 + 6i$ evaluate the following:

(i) $z - 3w$ **20 marks**

$$4 + 6i - 3(3 - 5i) = 4 + 6i - 9 + 15i$$
$$= -5 + 21i$$

(ii) $\frac{z}{2w}$ **20 marks**

$$= \frac{(4 + 6i)}{(6 - 10i)}\frac{(6 + 10i)}{(6 + 10i)}$$

$$= \frac{24 + 76i + 60i^2}{36 - 100i^2}$$

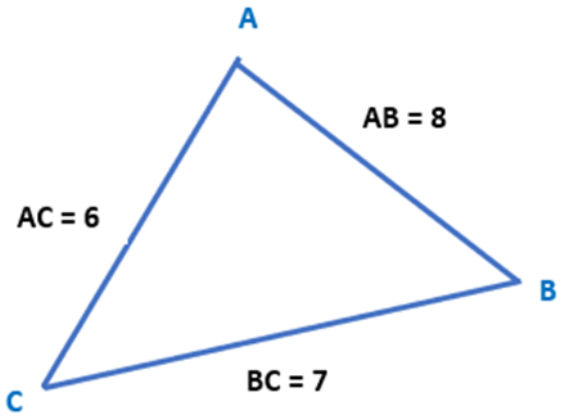
$$= \frac{24 + 76i - 60}{36 + 100}$$

$$= \frac{-36 + 76i}{136}$$

$$= \frac{-36}{136} + \frac{76i}{136}$$

$$\frac{-9}{34} + \frac{19i}{34}$$

10. Calculate the size of the angle at vertex A (angle CAB) in the triangle below. Give your answer correct to one decimal place, if necessary. **40 marks**



$$let \angle CAB = A$$
$$a = 7, \quad b = 6, \quad c = 8$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$7 = 6^2 + 8^2 - 2(6)(8)cosA$$

$$49 = 36 + 64 - 96\cos A$$

$$\cos A = \frac{36 + 64 - 49}{96} = 0.53125$$

$$A = \cos^{-1} 0.53125 = 57.9^0$$

Section B (200 Marks)
2 Structured Questions.

Answer ALL questions
100 marks each

1. (a) The equation of the line l is $y = 5x + 2$

(i) Find the slope of a line perpendicular to line l **10 marks**

$$m = -\frac{1}{5}$$

(ii) Find the equation of the line m perpendicular to line l and which passes through the point (-2, 1) **10 marks**

$$y - y1 = m(x - x1)$$
$$y - 1 = -\frac{1}{5}(x - (-2))$$
$$y - 1 = -\frac{1}{5}(x + 2)$$
$$5y - 5 = -x - 2$$
$$x + 5y - 3 = 0$$

(b) Draw a graph of $f(x) = x^3 + 3x^2 - 4x - 12$ **30 marks**

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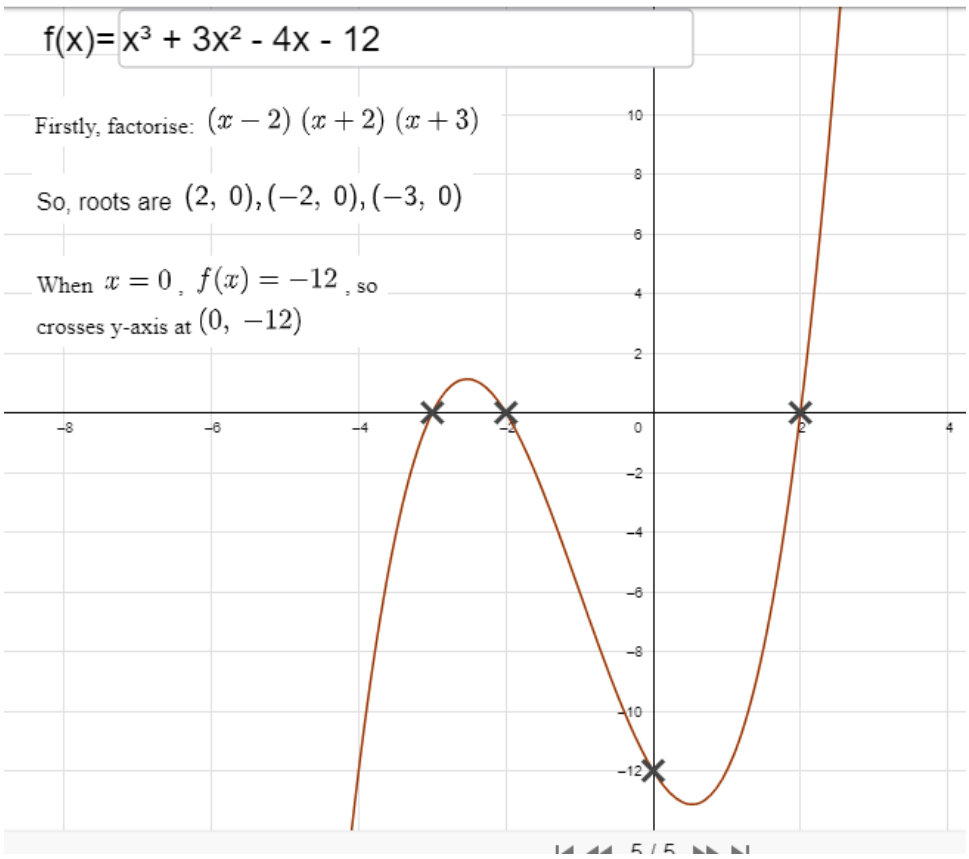
in the domain $\{-3 \ll x \ll 2\}, x \in R$

x	x^3	$3x^2$	-4x	-12	f(x)
-3	-27	27	12	-12	0
-2	-8	12	8	-12	0
-1	-1	3	4	-12	-6
0	0	0	0	-12	-12
1	1	3	-4	-12	-12
2	8	12	-8	-12	0

x	f(x)
-3	0
-2	0
-1	-6
0	-12
1	-12
2	0



Use the graph to write down the following

(i) Roots of the equation $f(x) = 0$ **10 marks**

$x = -3, -2, \quad 2$

$Points: (-3,0), (-2,0), (2,0)$

(ii) Find the coordinates of the local minimum point. (0.53,-13.1) **10 marks**

(iii Find the coordinates of the local maximum point. (-2.53, 1.13) **10 marks**

(iv The domain of values of x for which $f(x)$ is negative **10 marks**

$-2 < x < 2$

(v) The domain of values of x for which $f(x)$ is negative and increasing. **10 marks**

$0.53 < x < 2$

2. (a)

(i) Explain in your own words what is meant by the term 'mode' in relation to statistics? **10 marks**

Mode is the **value that appears most often** in a set of data values

(ii) Complete the cumulative frequency table below from the given grouped frequency distribution table.

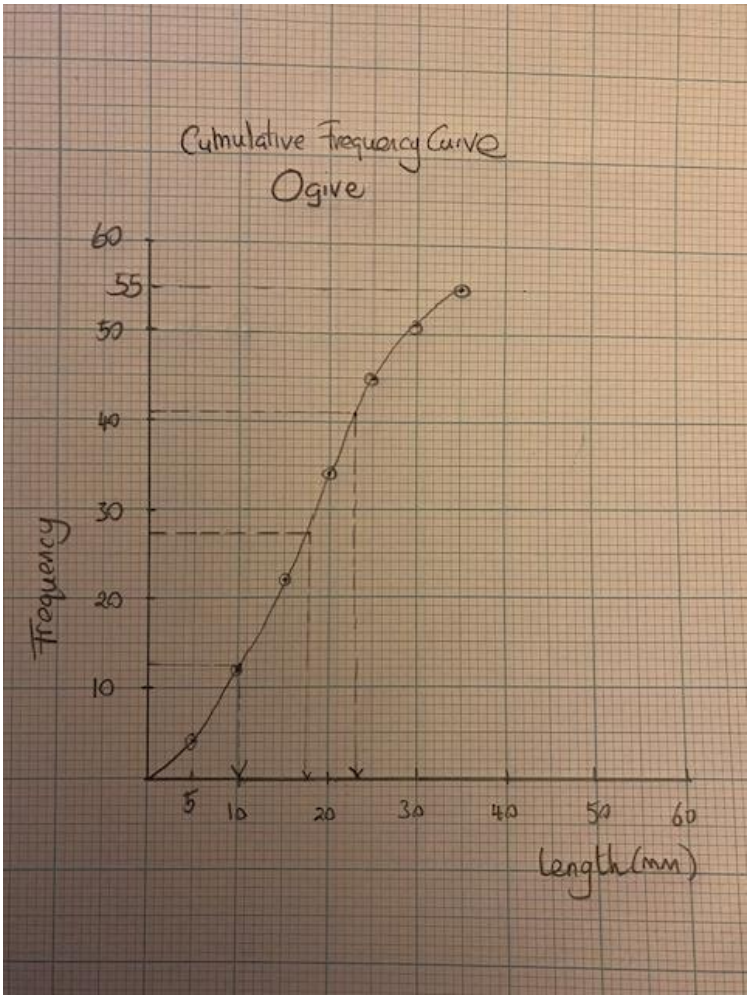
Length in mm (x)	1-5	6-10	11-15	16-20	21-25	26-30	31-35
Frequency (f)	4	8	10	12	11	6	4

Length in mm (x)	≤5	≤10	≤15	≤20	≤25	≤30	≤35
Frequency (f)	4	12	22	34	45	51	55

10 marks

Using the cumulative frequency table above, draw the cumulative frequency curve(ogive) on graph paper and use your graph to provide answers for the following:

10 marks



(iii) Interquartile range

10 marks

Upper quartile value at frequency of 41.25 (75% of 55) = approx. 23mm

Lower quartile value at frequency of 13.75 (25% of 55 = approx. 10mm

Interquartile range = 23-10 = 13mm

(iv) Median **10 marks**

Median – Value at middle frequency of 27.5 (50% of 55) – approx. 17.5mm

2. (b) A card is selected at random from a pack of 52 and then replaced. A second card is then selected. What is the probability that

(i) The first card is a heart = $\frac{13}{52} = \frac{1}{4}$ **10 marks**

(ii) Both cards are hearts = $\frac{13}{52} \cdot \frac{13}{52} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$ **10 marks**

(iii) The first card is red and the second card is black **10 marks**

$\frac{26}{52} \cdot \frac{26}{52} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(iv) The first card is a queen and the second card is black **10 marks**

$\frac{4}{52} \cdot \frac{26}{52} = \frac{1}{13} \cdot \frac{1}{2} = \frac{1}{26}$

(v) Neither card is a heart **10 marks**

$\frac{39}{52} \cdot \frac{39}{52} = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

Section C (200 Marks)
2 structured questions. Answer ALL 2.
100 marks each

3. (a) Differentiate with respect to x **30 marks**

$$y = \frac{2x^2 - 2x + 5}{x - 3}$$

$$quotient\ rule: \quad u = 2x^2 - 2x + 5 \quad v = x - 3$$

$$\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$\frac{du}{dx} = 4x - 2 \quad \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{(x-3).(4x-2) - (2x^2 - 2x + 5).1}{(x-3)^2}$$

$$\frac{dy}{dx} = \frac{4x^2 - 14x + 6 - 2x^2 + 2x - 5}{x^2 - 6x + 9}$$

$$\frac{dy}{dx} = \frac{2x^2 - 12x + 1}{x^2 - 6x + 9}$$

(b) Using integration, find the area bound by the curve $y = 2x^2 - x + 3$ the x *axis* and the lines $x = 2$ and $x = 4$ **30 marks**

$$\int_2^4 (2x^2 - x + 3)dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^2}{2} + 3x + c\right]_2^4$$

$$= \left[\frac{2(4^3)}{3} - \frac{(4)^2}{2} + 3(4) + c\right] - \left[\frac{2(2^3)}{3} - \frac{(2^2)}{2} + 3(2) + c\right]$$

$$= \left[\frac{128}{3} - \frac{16}{2} + 12 + c\right] - \left[\frac{16}{3} - 2 + 6 + c\right]$$

$$= \frac{112}{3}$$

$$= 37\frac{1}{3} units^2$$

(c) Find the turning points of the curve $y = \frac{x^3}{3} + \frac{x^2}{2} - 2x$ **20 marks**

and determine if they are minimum or maximum turning points **20 marks**

$$y = \frac{x^3}{3} + \frac{x^2}{2} - 2x$$

$$\frac{dy}{dx} = x^2 + x - 2$$

$$\frac{dy}{dx} = 0$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x = -2 \text{ or } x = 1$$

$$\text{when } x = -2 \quad y = \frac{10}{3}$$

$$\therefore \left(-2, \frac{10}{3}\right) \text{ is a turning point}$$

$$\text{when } x = 1 \quad y = -\frac{7}{6}$$

$$\therefore \left(1, -\frac{7}{6}\right) \text{ is also a turning point}$$

$$\frac{dy^2}{dx^2} = 2x + 1$$

$$\text{At } x = -2 \quad \frac{d^2y}{dx^2} = -3 \quad \text{which is negative} \quad \therefore \text{local max}$$

$$\therefore \left(-2, \frac{10}{3}\right) \text{ is a maximum turning point}$$

$$\text{At } x = 1 \quad \frac{d^2y}{dx^2} = 3 \quad \text{which is positive} \quad \therefore \text{local min}$$

$$\therefore \left(-2, \frac{10}{3}\right) \text{ is a minimum turning point}$$

4. **(a)** Solve for x and y in the following equation
$2(x + yi) = 4(2 + 3i) - 2(1 - 2i)$ **30 marks**

$$2x + 2yi = 8 + 12i - 2 + 4i$$
$$2x + 2yi = 6 + 16i$$
$$2x = 6 \ \therefore x = 3$$
$$2y = 16 \ \therefore y = 8$$

(b) Evaluate i^8 **20 marks**

$$= \sqrt{-1}^8$$
$$=(\sqrt{-1}.\sqrt{-1}).(\sqrt{-1}.\sqrt{-1}).(\sqrt{-1}.\sqrt{-1}).(\sqrt{-1})\ (\sqrt{-1})$$
$$= (-1)(-1)(-1))(-1) = 1$$

(c) Solve the complex equation $z^2 - 5z + 15 = 0$ **50 marks**
Write your answers in the form a+bi

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{5 \pm \sqrt{-5^2 - 4(1)(15)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 60}}{2}$$

$$= \frac{5 \pm \sqrt{-35}}{2}$$

$$= \frac{5 \pm \sqrt{35}\, i}{2}$$

$$z = \frac{5}{2} + \frac{\sqrt{35}}{2}\, i \quad or \quad z = \frac{5}{2} - \frac{\sqrt{35}}{2}\, i$$