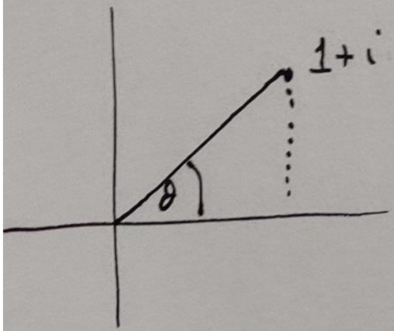
Solutions to Power Exercises

$(1+i)^6$

Write in polar form

Modulus $\Rightarrow \sqrt{(1)^2+(1)^2} = \sqrt{2}$



$\theta \Rightarrow \tan\theta = \frac{1}{1}$

$\theta = \frac{\pi}{4}$

Polar form

$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Using De Moivre's

$\therefore \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^6$

$\left(2^{1/2}\right)^6\left(\cos\frac{6\pi}{4} + i\sin\frac{6\pi}{4}\right)$

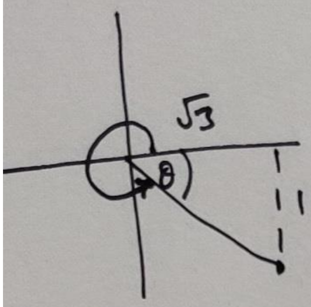
$2^3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$8\left(\cos 270^\circ + i\sin 270^\circ\right)$

$8\left(0 + i\,{-1}\right)$

$-8i$

$(\sqrt{3} - i)^9$



Modulus $\sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

Argument $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or 30°

We can use 330° or $-30^\circ, \frac{11\pi}{6}$

Polar Form $r(\cos\theta + i\sin\theta)$

$(2(\cos 330 + i\sin 330))^9$

$2^9(\cos 9(330) + i\sin 9(330))$

$512(\cos 2970 + i\sin 2970)$

$512(\cos 90^\circ + i\sin 90^\circ)$

$512(0 + i(1))$

$512(i)$

$512i$

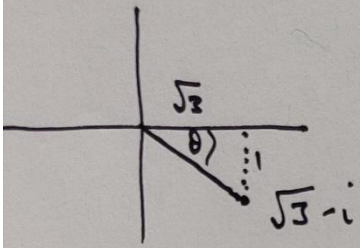
Note: $2970 - 360 = 2610$

$\vdots$

$810 - 360 = 450$

$450 - 360 = 90$

$(\sqrt{3} - i)^9$



Modulus $\quad \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

Argument $\quad \tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or 30° so we use $-\frac{\pi}{6}$ or -30°

Polar Form

$$r(\cos\theta + i\sin\theta)$$

$$\left(2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)\right)^9$$

$$2^9\left(\cos\left(-\frac{9\pi}{6}\right) + i\sin\left(-9\frac{\pi}{6}\right)\right)$$

$$512\left(\cos -\frac{3\pi}{2} + i\sin -\frac{3\pi}{2}\right)$$

$$512(\cos 270^\circ + i\sin -270^\circ) \text{ or } 512(\cos 270 - i\sin 270)$$

$$512(0 + i(-1))$$

$$512(i)$$

$$512i$$

Note:

$$\cos(-A) = \cos A$$

$$\sin(-A) = -\sin A$$

$$\tan(-A) = -\tan A$$