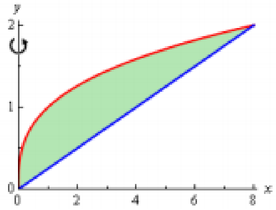
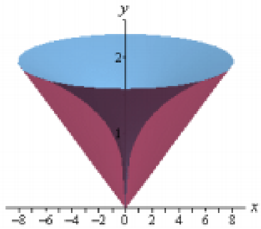
Volume when rotating about Y-Axis using Integration

Determine the volume of the solid generated by rotating the region bounded by $y = \sqrt[3]{x}$, and $y = \dfrac{x}{4}$ that lies in the first quadrant about the y-axis.

Solution

Step 1: Graph the bounding region and a graph of the object. The cross section is cut **perpendicular** to the axis of rotation and it is a horizontal washer. The inner and outer radii of the washer are x values, so we will need to rewrite our functions into the form $x = f(y)$.

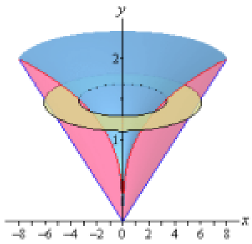


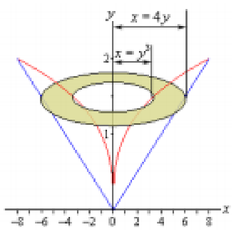


Here are the functions written in the correct form for this example.

$$y = \sqrt[3]{x} \Rightarrow x = y^3 \quad \text{and} \quad y = \frac{x}{4} \Rightarrow x = 4y$$

Step 2. Graph couple of sketches of the boundaries of the walls of this object as well as a typical washer. The sketch on the left includes the back portion of the object to give a little context to the figure on the right.





The cross-sectional area is then, $A(y) = \pi\left((4y)^2 - \left(y^3\right)^2\right) = \pi\left(16y^2 - y^6\right)$

Step 3. Working from the bottom of the solid to the top we can see that the first cross-section will occur at $y=0$ and the last cross-section will occur at $y=2$.These will be the limits of integration.

Step 4. The volume is then, $V = \int_c^d A(y)dy = \pi\int_0^2 \left(16y^2 - y^6\right)dy = \pi\left(\frac{16}{3}y^3 - \frac{1}{7}y^7\right)\Big|_0^2 = \frac{512\pi}{21}$