Find the area bounded by the curve $y=x^{3}$, the $y$-axis and the lines $y=-1$ and $y=8$ ?

Area bounded by a curve to the y-axis $\quad y=f_{a}^{\text {area }}=\int_{a}^{b} x(x) x d y$
Find the area bounded by the curve $y=x^{3}$, the $y$-axis and the lines $y=-1$ and $y=8$.


Area

$$
\begin{aligned}
A & =\frac{3}{4}\left[y^{4 / 3}\right]_{0}^{8} \\
& =\frac{3}{4}\left[(\sqrt[3]{8})^{4}-0\right] \\
& =12 \text { unit }^{2}
\end{aligned}
$$

B:

$$
\begin{aligned}
\int_{-1}^{0} y^{1 / 3} d y & =\frac{3}{4}\left[y^{4 / 3}\right]_{-1}^{0} \\
& =\frac{3}{4}\left[0-(\sqrt[3]{-1})^{4}\right] \\
& =-\frac{3}{4}
\end{aligned}
$$

$\therefore$ Area $B=\frac{3}{4}$ unit $^{2}$

Find the area bounded by the curve $y=\sqrt{x+1}$ the $y$-axis and the lines $y=1$ and $y=3$ ?

$(0)^{2}-1=x$

$$
-1=>i
$$



$$
\int_{1}^{3} y^{2}-1=\left|\frac{y^{3}}{3}-y+c\right|_{1}^{3}
$$

$$
\begin{aligned}
& \left(\frac{(3)^{3}}{3}-(3)+c\right)-\left(\frac{(1)^{3}}{3}-(1)+c\right) \\
& 9-3+\varphi-\frac{1}{3}+1-\not \subset \\
& 6^{2} / 3 \text { mils }
\end{aligned}
$$

Calculate the area of the region bounded by the curve $y=x^{2}$, the $y$-axis and the lines $y=1$ and $y=4$


$$
\begin{aligned}
& g=x^{2} \\
& x= \pm \sqrt{y} \\
& x= \pm y^{\frac{1}{2}}
\end{aligned}
$$

$$
\begin{aligned}
A & =\int_{1}^{4} y^{\frac{1}{2}} d y \\
& =\left[\frac{2}{3} x y^{\frac{3}{2}}\right]_{1}^{4} \\
& =\frac{2}{3} \times 4^{\frac{3}{2}}-\frac{2}{3} \times 1^{\frac{3}{2}} \\
& =\frac{14}{3} \text { units }^{2}
\end{aligned}
$$



$$
\begin{aligned}
\text { Area } & =\int_{-1}^{3}\left(x^{2}+1\right) d x \\
& =\left[\frac{x^{3}}{3}+x\right]_{-1}^{3} \\
& =\left[\frac{(3)^{3}}{3}+3\right]-\left[\frac{(-1)^{3}}{3}+(-1)\right] \\
& =12+\frac{1}{3}+1 \\
& =\frac{40}{3} \text { sq. wits }
\end{aligned}
$$



Area $A=\int_{1}^{3}\left(4 x^{2}-x^{3}-3 x\right) d x$

$$
\begin{aligned}
& =\left[\frac{4 x^{3}}{3}-\frac{x^{4}}{4}-\frac{3 x^{2}}{2}\right]_{1}^{3} \\
& =\left[\frac{4}{3}(3)^{3}-\frac{(3)^{4}}{4}-\frac{3(3)^{2}}{2}\right] \\
& -\left[\frac{4(1)^{3}}{3}-\frac{(1)^{4}}{4}-\frac{3(1)^{2}}{2}\right] \\
& =36-\frac{81}{4}-\frac{27}{2}-\frac{4}{3}+\frac{1}{4}+\frac{3}{2} \\
& =\frac{8}{3} \text { sq. units }
\end{aligned}
$$



Area $A=\int_{1}^{3}\left(4 x^{2}-x^{3}-3 x\right) d x$

$$
\begin{aligned}
= & {\left[\frac{4 x^{3}}{3}-\frac{x^{4}}{4}-\frac{3 x^{2}}{2}\right]_{1}^{3} } \\
= & {\left[\frac{4}{3}(3)^{3}-\frac{(3)^{4}}{4}-\frac{3(3)^{2}}{2}\right] } \\
& -\left[\frac{4(1)^{3}}{3}-\frac{(1)^{4}}{4}-\frac{3(1)^{2}}{2}\right] \\
= & 36-\frac{81}{4}-\frac{27}{2}-\frac{4}{3}+\frac{1}{4}+\frac{3}{2}
\end{aligned}
$$

$$
\begin{aligned}
\int_{0}^{1}\left(4 x^{2}-x^{3}-3 x\right) d x & =\left[\frac{4 x^{3}}{3}-\frac{x^{4}}{4}-\frac{3 x^{2}}{2}\right]_{0}^{1} \\
& =\frac{4(1)^{3}}{3}-\frac{(1)^{4}}{4}-\frac{3(1)^{2}}{2}-0 \\
& =-\frac{5}{12} \quad \begin{array}{l}
\text { Turns out negative } \\
\text { when under the } x \text {-axis }
\end{array}
\end{aligned}
$$

$\therefore$ Area $B=\frac{5}{12}$ sq. units
Area $B=-\int_{0}^{1} y d x$

$$
\begin{aligned}
\therefore \text { Total Area } & =\text { Area } A+B \\
& =\frac{8}{3}+\frac{5}{12} \\
& =\frac{37}{12} \text { sq. units }
\end{aligned}
$$

