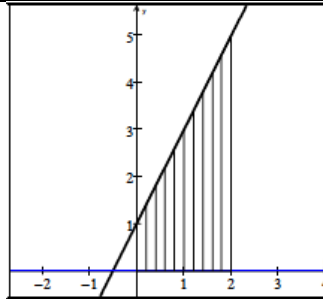**Steps for finding the Area Under a Curve**

- **Graph** $f(x)$

- **Shade the region enclosed by** $f(x)$; $x = a$; $x = b$; **and the** x-**axis. You can only take the area of a closed region, so you must include the** x-**axis** ($y = 0$)

- **As long as the entire shaded region is above the** x-**axis then** $\boxed{\text{Area} = \int_a^b f(x)\,dx}$
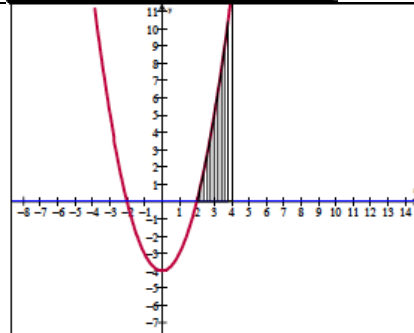
Examples:

1) $f(x) = 2x + 1$; $a = 0$, $b = 2$

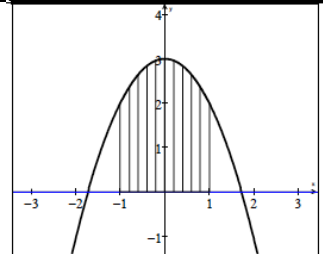$$\boxed{\int_0^2 (2x+1)\,dx = x^2 + x + C\Big|_0^2 = (4+2) - 0 = \boxed{6}}$$

2) $f(x) = x^2 - 4$; $a = 2$; $b = 4$

$$\boxed{\int_2^2 (x^2 - 4)\,dx = \frac{x^3}{3} - 4x + C\Big|_2^4 = \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right) = \boxed{\frac{32}{3}}}$$

3) $f(x) = 3 - x^2$; $a = -1$; $b - 1$

$$\boxed{\int_{-1}^1 (3 - x^2)\,dx = 3x - \frac{x^3}{3} + C\Big|_{-1}^1 = \left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right) = \boxed{\frac{16}{3}}}$$
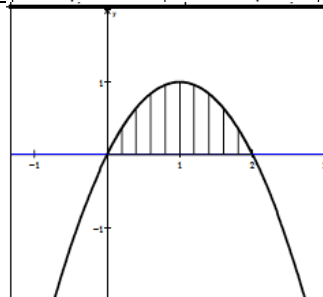

4) $f(x) = 2x - x^2$; $a = 0$, $b = 2$
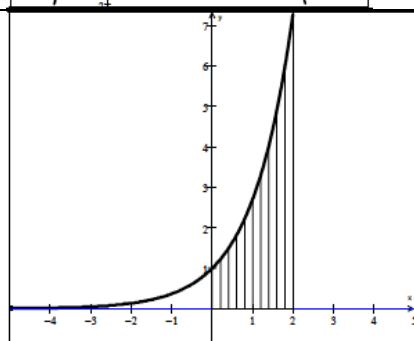
Make graphing easy!! Find the x-**intercepts:**

$2x - x^2 = 0 \rightarrow x(2 - x) = 0 \rightarrow x = 0$ and $x = 2$

$$\boxed{\int_0^2 (2x - x^2)\,dx = x^2 - \frac{x^3}{3}\Big|_0^2 = \left(4 - \frac{8}{3}\right) - 0 = \boxed{\frac{4}{3}}}$$

5) $f(x) = e^x$; $a = 0$, $b = 2$

$$\boxed{\int e^x\,dx = e^x + C\Big|_0^2 = e^2 - e^0 = \boxed{e^2 - 1}}$$

Problems #1 – 8: <u>Graph</u> and <u>find the area</u> under the graph of $f(x)$ from *a* to *b* by integrating.

1. $f(x) = x + 1$; $a = 0$, $b = 3$	2. $f(x) = 4 - x$; $a = -1$, $b = 2$
3. $f(x) = 4 - x^2$; $a = -2$, $b = 2$	4. $f(x) = 4x - x^2$; $a = 0$, $b = 4$
5. $f(x) = \cos x$; $a = -\dfrac{\pi}{2}$, $b = \dfrac{\pi}{2}$	6. $f(x) = \sin x$; $a = \dfrac{\pi}{6}$, $b = \dfrac{\pi}{3}$
7. $f(x) = e^{2x}$; $a = 0$, $b = 1$	8. $f(x) = e^x$; $a = -1$, $b = 1$

For #9 – 10: <u>Graph</u> and <u>find the area</u> of the region bounded by $f(x)$, the *x*-axis, and the values of *a* and *b*.
In these two problems, you need to "find" *a* (left most *x*-value) and *b* (right most *x*-value). They are where $f(x)$ crosses the *x*-axis.

9. Bounded by the *x*-axis and the parabola $y = 4 - x^2$ (What is *a*?   *b*?   )
10. Bounded by the *x*-axis and the parabola $y = 4x - x^2$ (What is *a*?   *b*?   )

For #11: <u>Graph</u> the region stated and then find the area of the bounded region.

| |
|---|
| 11. Bounded by the curve $y = \sqrt{x}$ and the lines $x = 4$ and $y = 0$ (What is *a*?   *b*?   ) |

#12 and #13 are a little trickier because the region bounded does not involve the *x*-axis.
For these problems, you must:
- Graph the given functions to find the enclosed region that you will find the area of
- Write down: *Top function - Bottom function* (in terms of *x* only)
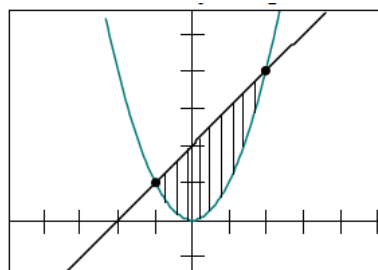- Find the values for *a* and *b* (A little Algebra)

- **Integrate to find area:** $\text{Area} = \int_a^b (Top - Bottom)\,dx$

---

12. Lying in the first quadrant and bounded by the curves $y = \sin x$, $y = 1$, and $x = 0$
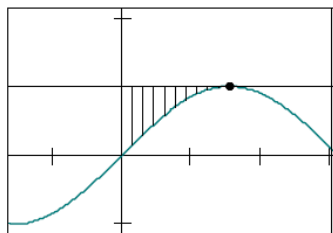
- What function is on *Top* of the shaded region?   On the *Bottom?*
- What is *Top – Bottom?*
- What is *a?*   *b?*
- Write the appropriate integral and find the area.

---

13. Bounded by the parabola $y = x^2$ and the line $y = x + 2$

- What function is on *Top* of the shaded region?   On the *Bottom?*
- What is *Top – Bottom?*
- What is *a?*   *b?*
- Write the appropriate integral and find the area.

Answers:

				5. 2			
1. $\dfrac{15}{2}$	2. $\dfrac{21}{2}$	3. $\dfrac{32}{3}$	4. $\dfrac{32}{3}$		6. $\dfrac{\sqrt{3}-1}{2}$	7. $\dfrac{e^2-1}{2}$	8. $\dfrac{e^2-1}{e}$
9. $a=2; b=-2$ $\dfrac{32}{3}$	10. $a=0; b=4$ $\dfrac{32}{3}$	11. $a=0; b=4$ $\dfrac{16}{3}$	12. $a = 0; b = \dfrac{\pi}{2}$ $Area = \dfrac{\pi}{2} - 1$	13. $y = x+2; y = x^2$ $x+2-x^2$ $a = -1; b = 2$ $Area = \dfrac{9}{2}$	14. $\dfrac{-3}{2}$	15. 0	