Graphs & Functions

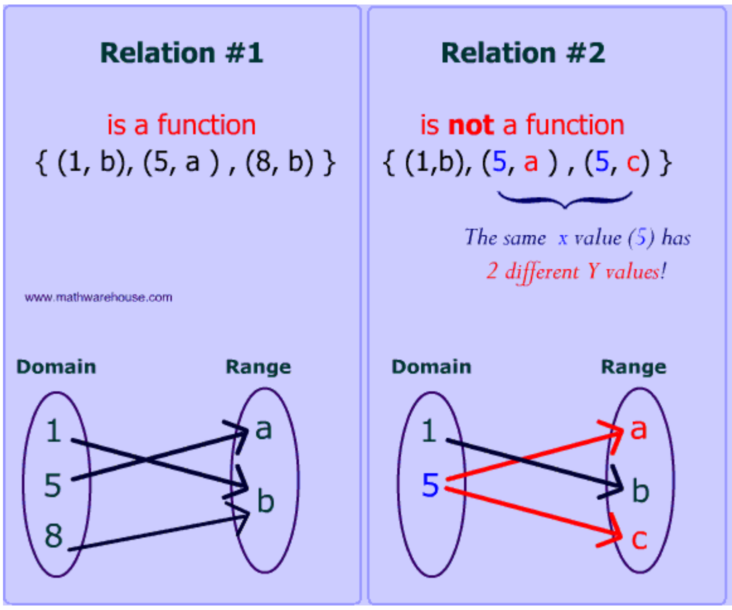
Learning Outcomes

1. Describe the properties of basic mathematical functions to include linear, quadratic, exponential, log and trigonometric functions
2. Define the inverse of a function
3. Graph linear and quadratic functions showing the relationship between the domain and range
4. Derive the inverse of a function from its algebraic expression
5. Calculate the equation of a straight line using a range of formulae to include distance between two points, slope, parallel lines and perpendicular lines
6. Solve maximum and minimum problems with limitations given by linear inequalities from graphs of linear inequalities and half planes
7. Analyse graphs of linear and quadratic functions for important properties to include domain and range, maximum and minimum values, increasing and decreasing intervals, periodicity

What is a function?

A function is a special relationship where each input has a single output. It is often written as "f(x)" where x is the input value. Example: $f(x)=\frac{x}{2}$ *("f of x equals x divided by 2").* It is a function because each input "x" has a single output $\frac{x}{2}$:

Eg : $f(2)=\frac{2}{2}=1.$



Input (Domain) ->	Function ->	**Output** (Range)
2	$f(x)=\frac{x}{2}$	1

Function Notation

Function notation is used to name functions for easy reference. Imagine if every function in the world had to start off with y =. Pretty soon, you would become confused about which y = you were talking about. You need some other way of naming things. Hence, we have function notation. $f(x)=, g(x)=, etc.$

Function Definition

$f(x) = 3x + 2$

$g(x, y) = x^2 + 3y$

In this example, the f is a function of x. That is, x is the independent variable, and the value of f depends on what x is. Also, g is a function of both x and y. The notation f(x) does not mean f times x. It means the "value of f evaluated at x" or "value of f at x" or simply "f of x".

Function Evaluation

$f(3) = 3(3) + 2 = 9 + 2 = 11$

$f(3)$ does not mean f times 3. It means the "value of f evaluated when x is 3".

$f(t) = 3(t) + 2 = 3t + 2$

Whatever is in parentheses on the left side of the function (t in this case) is substituted for the value of the independent variable on the right side.

$f(x + h) = 3(x + h) + 2 = 3x + 3h + 2$

Every occurrence of the independent variable is replaced by the quantity in parentheses. A common mistake is to take a quantity and apply linear transformations to it.

$f(x + h)$ *does not equal* $f(x) + h = 3x + 2 + h$

$f(x + h)$ *does not equal* $f(x) + f(h) = 3x+2 + 3h + 2 = 3x + 3h + 4$

It <u>does</u> equal $3(x + h) + 2 = 3x + 3h + 2$

$f(3x)$ *does not equal* $3 * f(x) = 3(3x+2) = 9x + 6$

It <u>does</u> equal $3(3x) + 2 = 9x + 2$

You also specify which function you want to use when you use function notation. Consider:

$g(x, y) = x^2 + 3y$

$g(2, 1) = (2)^2 + 3(1) = 4 + 3 = 7$

Since the order of the independent variables in the original definition was x and then y, the function g is evaluated when $x = 2$ and $y = 1$.

The notation $y = f(x)$ means: *'the value of y depends on the value of x'*. Hence, y and $f(x)$ are interchangeable, and the Y axis can also be called the $f(x)$ axis.

Note: When graphing a function it is very important not to draw the graph outside the given domain (i.e., the given values of x).

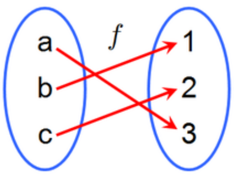
Exercises

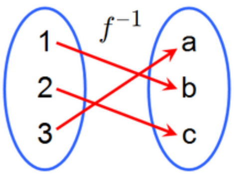
If f(x) = 2x-1 and g(x)=4x find the solutions to the following:

1. f(3)
2. f(-4)
3. f(0)
4. f(c)
5. g(5)
6. g(-1)
7. g(0)
8. g(a)
9. fg(1)
10. gf(2)
11. fg(x)
12. gf(x)

Inverse Function

In mathematics, an inverse function (or anti-function) is a function that "reverses" another function: if the function f applied to an input x gives a result of y, then applying its inverse function g to y gives the result x, and vice versa, i.e., f(x) = y if and only if g(y) = x.





Calculating the inverse function

Function	Evaluating Inverse Function
$x \to 3x+2$	$x \to 3x+2$
	$x-2 \to 3x$
	$\frac{x-2}{3} \to x$
	$f^{-1}(x) = \frac{x-2}{3}$

Function	Evaluating Inverse Function
$x \to 4x-15$	$x \to 4x-15$
	$x + 15 \to 4x$
	$\frac{x+15}{4} \to x$
	$f^{-1}(x) = \frac{x+15}{4}$

Exercises

1. Find the inverse function of $f(x)$. State the domain of the inverse function $f^{-1}(x)$.

   (a) $f(x) = 2x + 4$

   (b) $f(x) = 2(x + 4)$

   (c) $f(x) = 3(x - 7) + 5$

   (d) $f(x) = x^2 - 4$

   (e) $f(x) = x^3 + 7$

   (f) $f(x) = (x + 7)^3$

Solutions

(a) The inverse function of $f(x) = 2x + 4$ is $f^{-1}(x) = \dfrac{x-4}{2}$.
The function $f^{-1}(x)$ has domain $(-\infty, \infty)$.

(b) The inverse function of $f(x) = 2(x + 4)$ is $f^{-1}(x) = \dfrac{1}{2}(x - 8)$.
The function $f^{-1}(x)$ has domain $(-\infty, \infty)$.

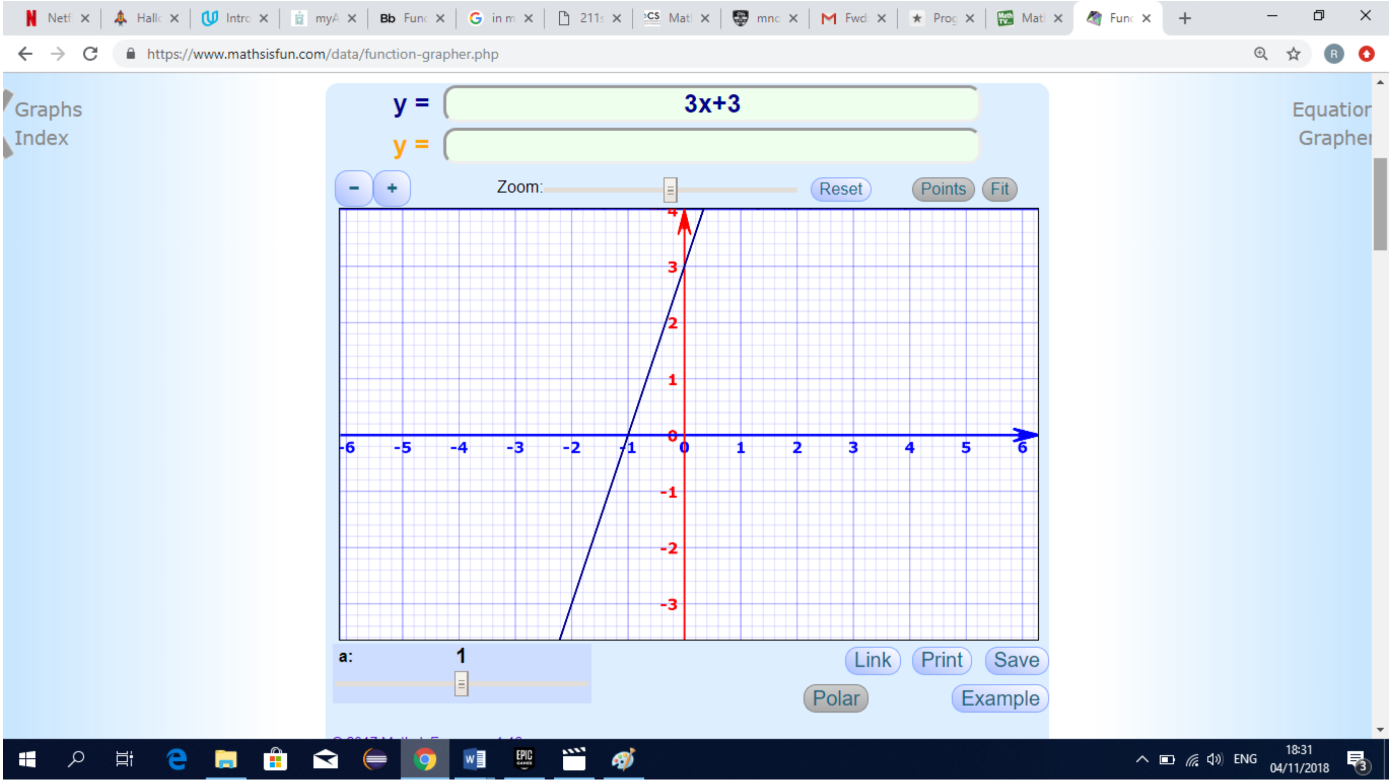
(c) The inverse function of $f(x) = 3(x - 7) + 5$ is $f^{-1}(x) = \dfrac{x-5}{3} + 7$.
The function $f^{-1}(x)$ has domain $(-\infty, \infty)$.

(d) The inverse function of $f(x) = x^2 - 4$ is $f^{-1}(x) = \sqrt{x+4}$. In order to make $f(x)$ one-to-one, so that there is no ambiguity in $f^{-1}(x)$, we need to restrict the domain of $f(x)$ to $[0, \infty)$. This means that the range of $f^{-1}(x)$ is also $[0, \infty)$.
The function $f^{-1}(x)$ has domain $[-4, \infty)$.

(e) The inverse function of $f(x) = x^3 + 7$ is $f^{-1}(x) = \sqrt[3]{x-7}$.
The function $f^{-1}(x)$ has domain $(-\infty, \infty)$.

(f) The inverse function of $f(x) = (x + 7)^3$ is $f^{-1}(x) = \sqrt[3]{x} - 7$.
The function $f^{-1}(x)$ has domain $(-\infty, \infty)$.

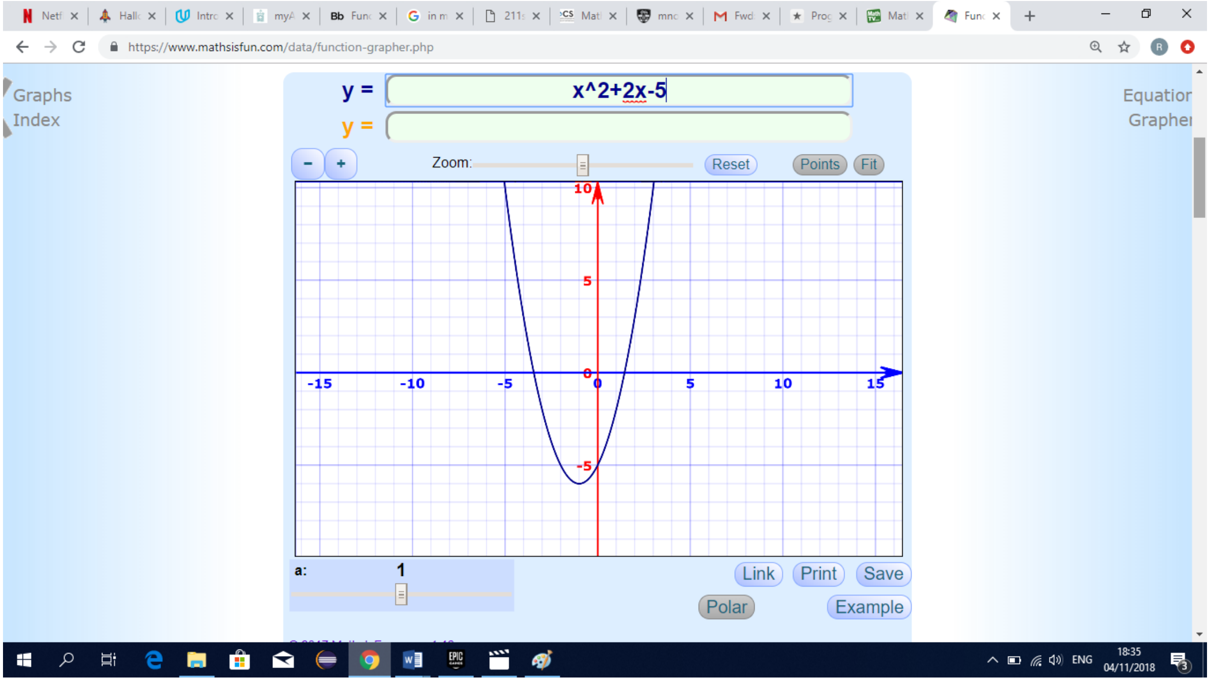
Linear Functions - https://www.mathsisfun.com/data/function-grapher.php



The above function has index of 1, subtract 1 from this which gives 0. This is a line with 0 turns, hence it must be a straight line(eg. A Linear function).

Quadratic Functions

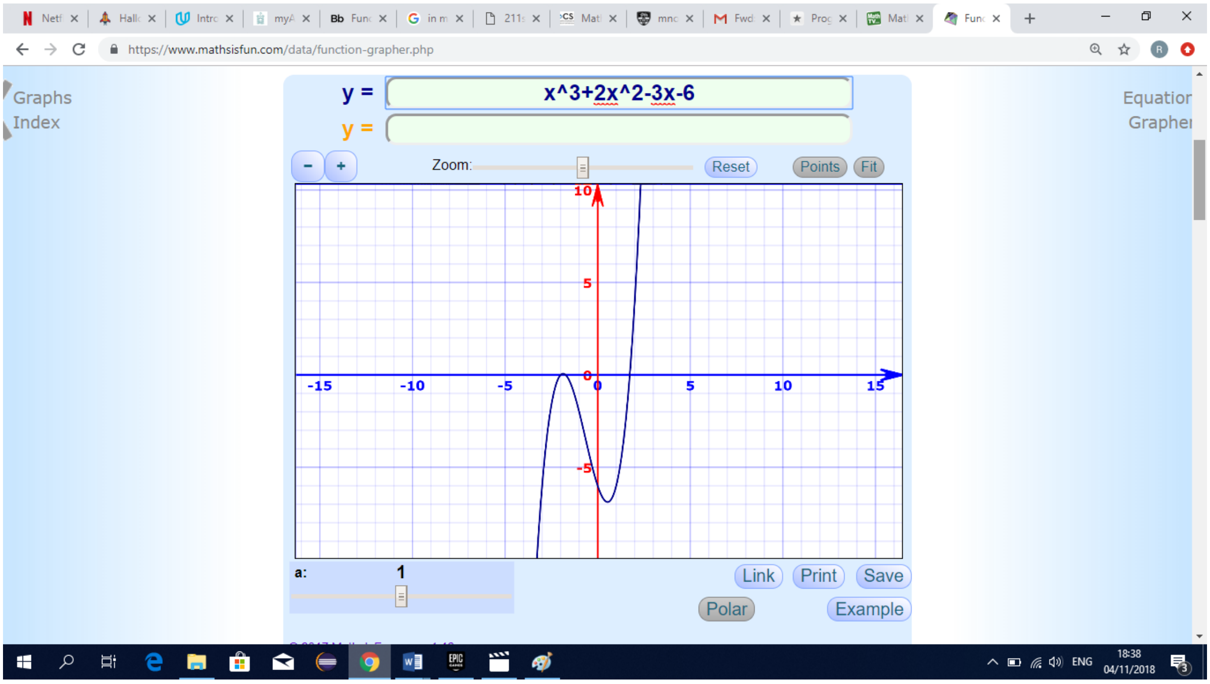
$$f(x) = x^2 + 2x - 5$$



The above function has an index of 2, subtract 1 from this which gives 1. This is a line with 1 turn, hence it must be a "U" or "n" shape as shown (eg. A Quadratic function). Note: A positive coefficient of x^2 *gives a u shape and a negative coefficient of* x^2 *gives a n shape.*

Cubic Functions

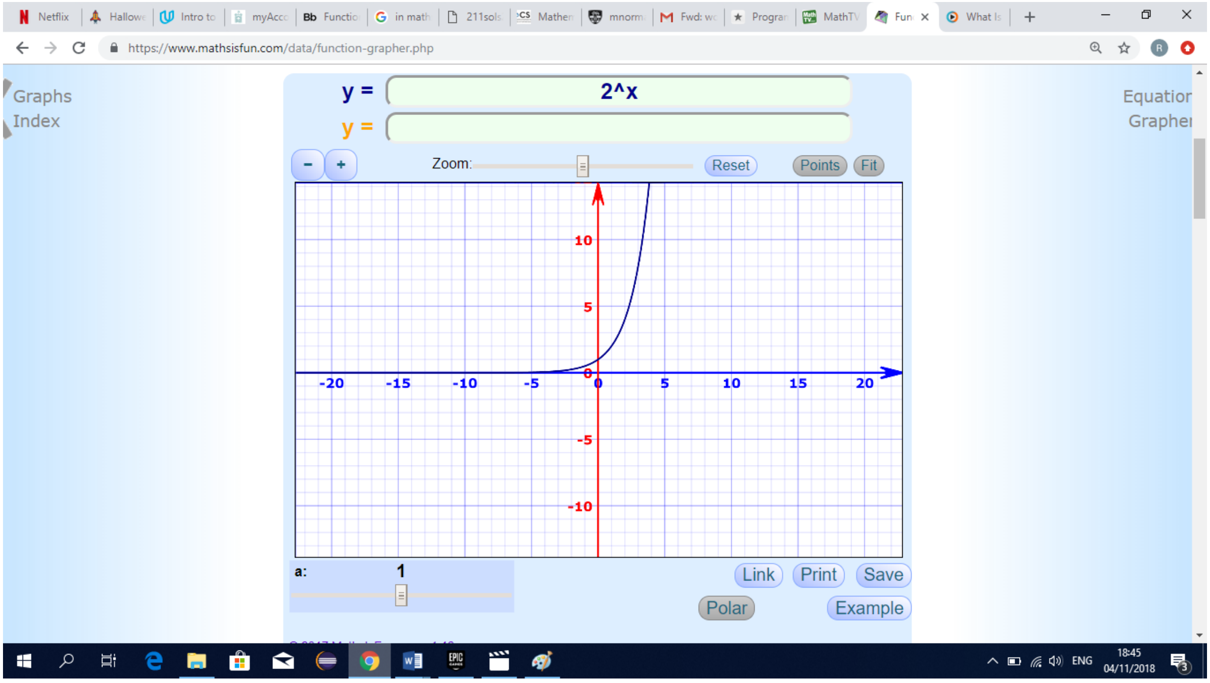
$$f(x) = x^3 + 2x^2 - 3x - 6$$



The above function has an index of 3, subtract 1 from this which gives 2. This is a line with 2 turns, hence it must be a sideways "S" shape as shown (eg. A Cubic function).

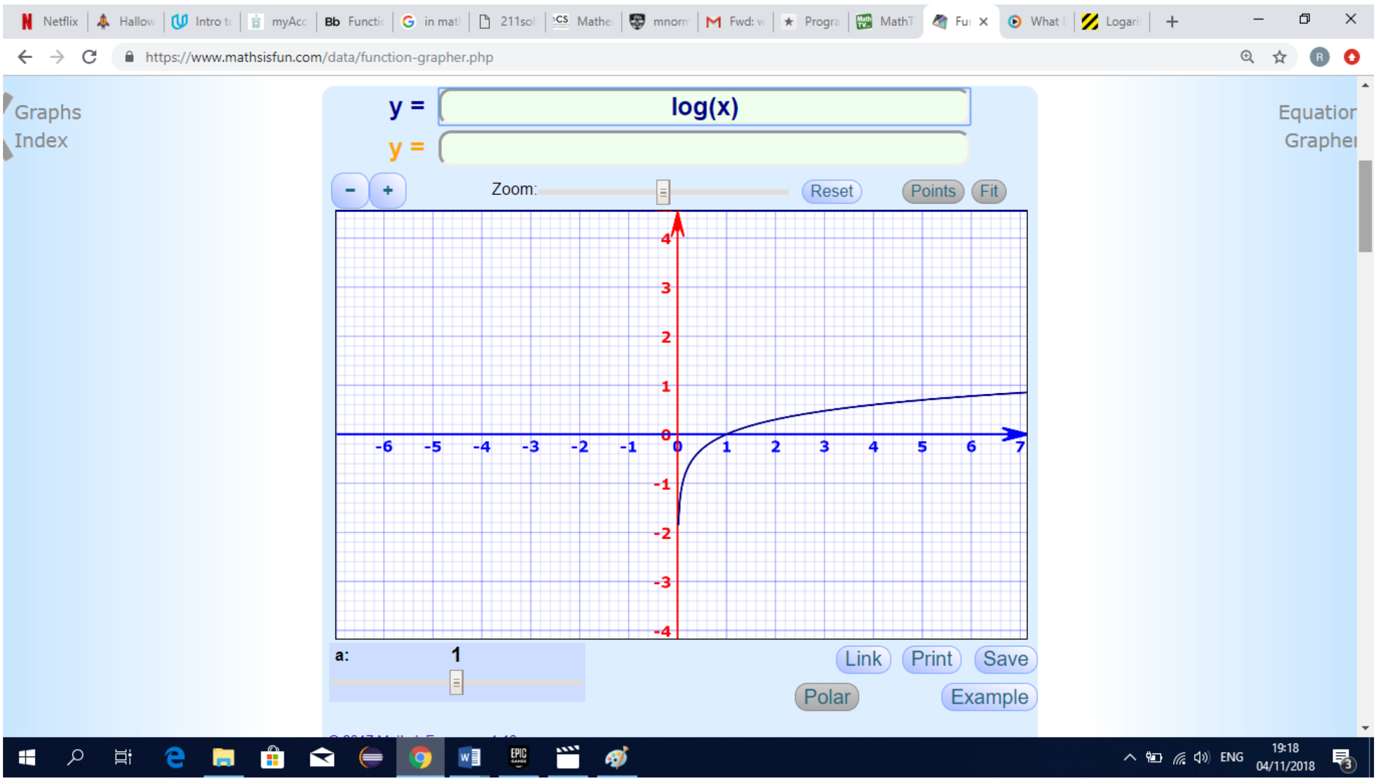
Exponential Function

The graph of $y = x^2$, is a function with an exponent. But it's not an exponential function. In an exponential function, the independent variable, or x-value, is the exponent, while the base is a constant. For example, $y = 2^x$ would be an exponential function.



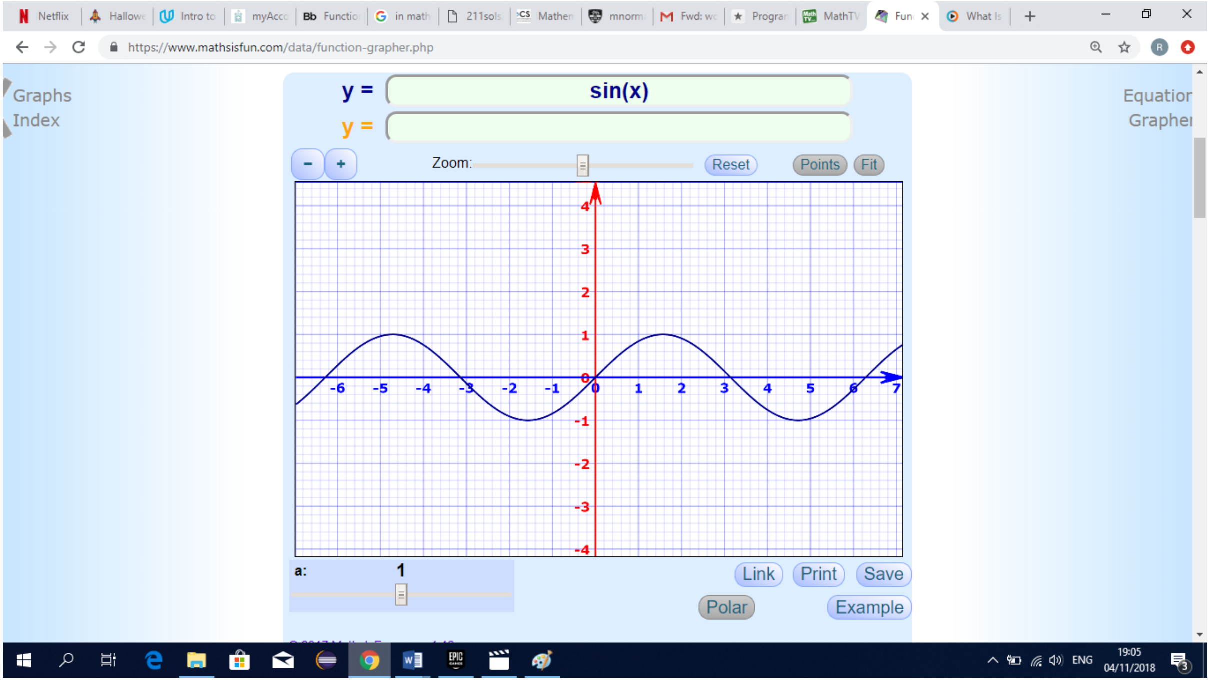
Logarithmic functions

Logarithmic functions are the inverses of exponential functions, and any exponential function can be expressed in logarithmic form. Similarly, all logarithmic functions can be rewritten in exponential form. Logarithms are really useful in permitting us to work with very large numbers while manipulating numbers of a much more manageable size. The word logarithm, abbreviated log, is introduced to satisfy this need. y = (the power on base 2) to equal x. This equation is rewritten as $y = \log_2 x$.

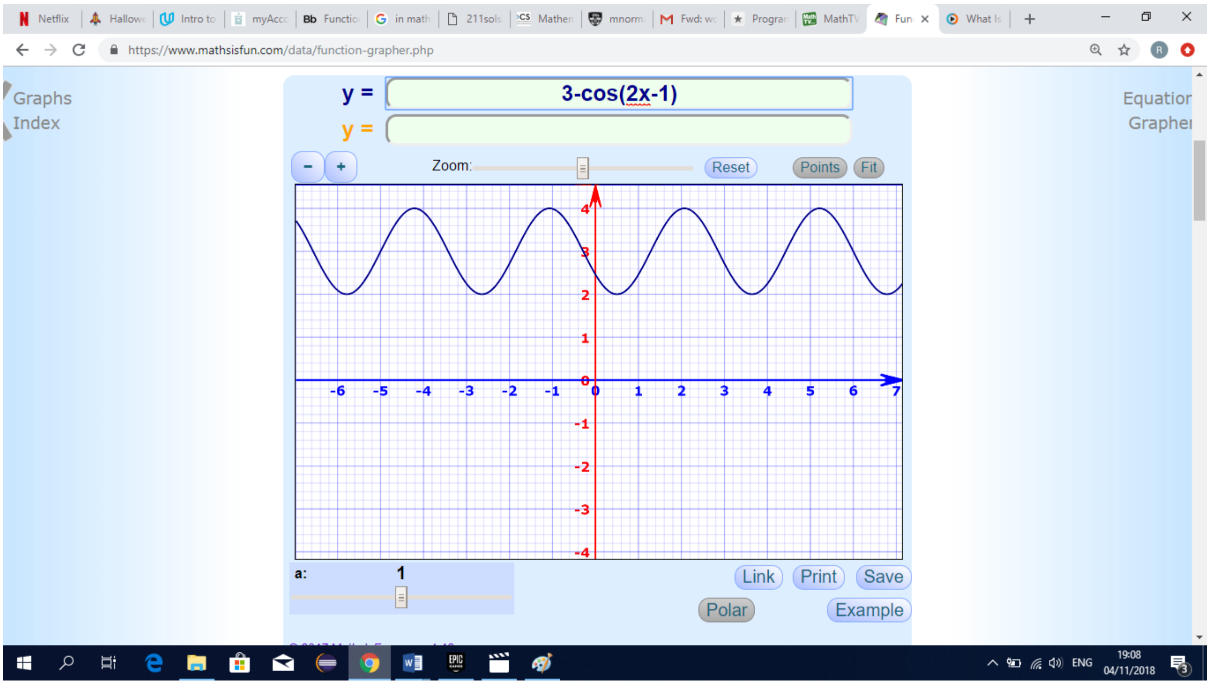


Trigonometric Functions

$$y \ or \ f(x) = sinx$$



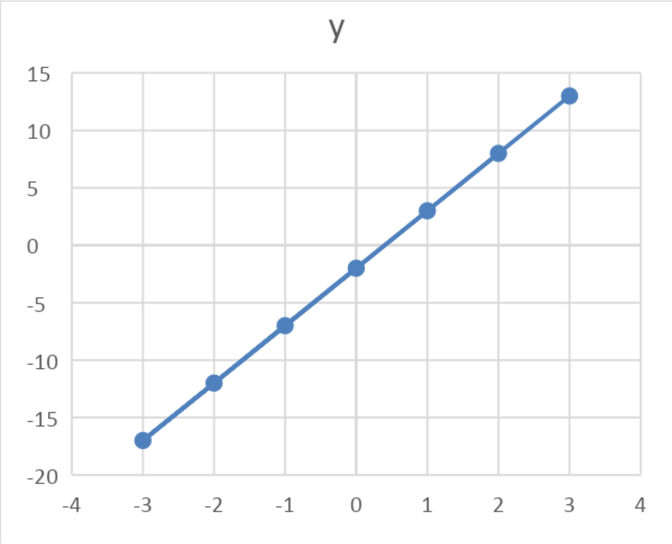
$$f(x)=3-\cos(2x-1)$$



Example 1 – Linear Function

Draw a graph of the function f(x) = 5x - 2 in the domain {-3 < x < 3}

x	-3	-2	-1	0	1	2	3
5x	-15	-10	-5	0	5	10	15
-2	-2	-2	-2	-2	-2	-2	-2
y	-17	-12	-7	-2	3	8	13

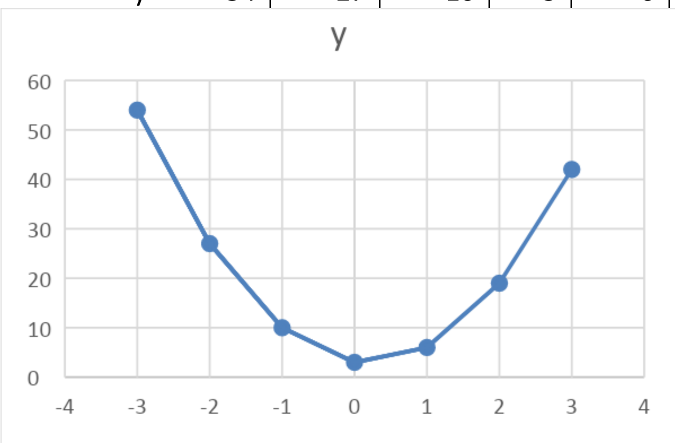


- Domain is {-3,-2,-1,0,1,2,3}
- Range is {-17,-12,-7,-2,3,8,13}

Example 2 – Quadratic Function

Draw a graph of the function f(x) = 5x^2 - 2x+3 in the domain {-3 < x < 3}

x	-3	-2	-1	0	1	2	3
5x^2	45	20	5	0	5	20	45
-2x	6	4	2	0	-2	-4	-6
3	3	3	3	3	3	3	3
y	54	27	10	3	6	19	42



- Domain is {-3,-2,-1,0,1,2,3}
- Range is {54,27,10,3,6,19,42}

2. Find the range of each of these functions:

(i) $f(x) = 2x,\ x \in \{2, 3, 4\}$ (ii) $f(x) = 2x - 3,\ x \in \{-1, 0, 1\}$

(iii) $f(x) = 2x^2,\ x \in \{-1, 0, 1, 2\}$ (iv) $f(x) = 2x - 1,\ x \in \{-2, 0, 2\}$

3. Say if each of the following relations is a function:

(i) $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$

(ii) $\{a, b), (b, c), (a, d), (d, b)\}$

4. Given that $f(x) = 3x - 4$, find the values of

(i) $f(4)$ (ii) $f(-3)$ (iii) $f(k)$ (iv) $f(2x)$

5. $f: x \to 2x + 1$ and $g: x \to x^2$ are two functions.

What is (i) $f(4)$ (ii) $g(-3)$ (iii) $fg(2)$ (iv) $gf(4)$?

6. f and g are two functions such that

$f(x) = 2x - 1$ and $g(x) = 3x + 2$.

Find (i) $fg(1)$ (ii) $gf(-3)$ (iii) $gf(x)$ (iv) $fg(x)$.

7. The functions f and g are defined as follows:

$f: x \to 2x - 1$ and $g: x \to x^2 + 2$

Find (i) $fg(-2)$ (ii) $gf(\frac{1}{2})$ (iii) $fg(x)$ (iv) $gf(x)$.

For what values of x is $gf(x) = fg(x)$?

8. $f(x) = 2x^2$ and $g(x) = 3x - 1$ are two functions.

(a) Evaluate (i) $fg(\frac{1}{3})$ (ii) $gf(-2)$

(b) For what value(s) of x is

(i) $f(x) = g(x)$ (ii) $fg(x) = gf(x)$?

9. (i) The function f is defined by $f: R \to R: x \to 2x - 3$.

For what value of k is $f(k) + f(2k) = 0$?

(ii) The function g is defined by $g: x \to 7 - 3x$.

If $kg(-2) = g(24)$, find k.

(iii) Given the function $g: x \to x^2 - x$

For what values of t is $g(t) = g(2t)$?

10. The functions f and g are defined as follows:

$f: x \to 1 - 3x$ and $g: x \to x^2 - 1$.

(i) Find $fg(3)$ (ii) If $fg(x) = 3$, find x.

11. (a) f defines a set of couples, where $f: x \to \pm\sqrt{x}$.

Find (i) $f(9)$ (ii) $f(25)$.

Now state whether or not f is a function.

(b) Given that $f(x) = 3x + 2$, show that $f(x) + f(y) = f(x + y) + 2$.

12. (a) Two functions f and g are defined by

$f: x \to \dfrac{3}{x}$ and $g: x \to 4 - x$

(i) Find two values of x for which $gf(x) = fg(x)$

(b) If $f(x) = x^2$ and $g(x) = 4x - 3$, find the value of $[(f(3) \times g(3)] + fg(3)$.

1. Find the function $f^{-1}(x)$, given $f(x)$ in each of the following:

(i) $f(x) = x + 2$ (ii) $f(x) = 2x - 1$ (iii) $f(x) = 3x - 2$

(iv) $f(x) = 5x - 1$ (v) $f(x) = 3 - 4x$ (vi) $f(x) = \frac{1}{2}x - 4$

(vii) $f(x) = \dfrac{x - 2}{3}$ (viii) $f(x) = \dfrac{3x - 1}{2}$ (ix) $f(x) = \dfrac{5x - 2}{3}$

2. If f is the function $f(x) = 3x - 4$, find

(i) $f^{-1}(x)$ (ii) $f^{-1}(2)$ (iii) $f^{-1}(4)$ (iv) $f^{-1}(\frac{1}{2})$

3. If $f(x) = 2x + 4$ find

(i) $f(2)$ (ii) $f^{-1}(x)$ (iii) $f^{-1}(-2)$

Find also the value of x for which $f(x) = f^{-1}(x)$.

4. The function f is defined by $f: x \rightarrow \dfrac{2x + 5}{3}$.

Find (i) $f^{-1}(x)$ (ii) $f^{-1}(3)$

Show that $f^{-1}(3)$ is a solution of the equation $f(x) = 3$.

5. Two functions f and g are defined as follows:

$f: x \rightarrow 5x - 1$ and $g: x \rightarrow 3x + 5$.

Find (i) $f^{-1}(x)$ (ii) $g^{-1}(x)$.

Find also the value of k such that $f^{-1}(k) = g^{-1}(k)$.

https://mathbitsnotebook.com/Algebra1/Functions/FNNotationEvaluation.html

Graphing Linear Functions Video

Excel worksheets with examples of Linear and Quadratic Functions and Graphs – see Linear_and_Quadratic_Functions_and_Graphs file on website.

Graphs creator(linear/quadratic/trig/etc) - https://www.mathsisfun.com/data/function-grapher.php

Linear Functions

Example

Graph the function $g : x \rightarrow 2x + 1$, in the domain $-3 \leq x \leq 2, x \in \mathbf{R}$.

Solution

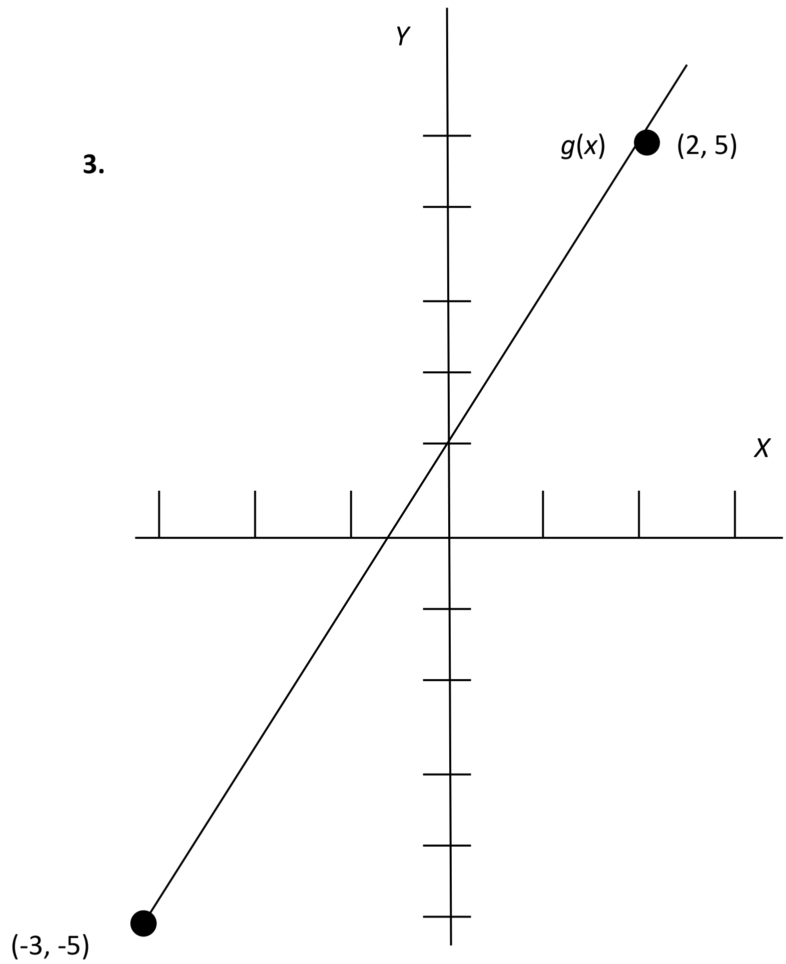
Let $y = g(x) \Rightarrow y = 2x + 1$.

1. Let $x = -3$ and $x = 2$

2. $y = 2x + 1$

$x = -3$	$x = 2$
$y = 2(-3) + 1$	$y = 2(2) + 1$
$y = -6 + 1$	$y = 4 + 1$
$y = -5$	$y = 5$

3.



Questions

Graph each of the following functions in the given domain ($x \in \mathbf{R}$ in each case):

1. $f : x \rightarrow x + 3$ in the domain $-4 \leq x \leq 2$
2. $g : x \rightarrow x - 2$ in the domain $-3 \leq x \leq 3$
3. $h : x \rightarrow 2x + 3$ in the domain $-2 \leq x \leq 4$
4. $g : x \rightarrow 3x - 2$ in the domain $-3 \leq x \leq 5$
5. $f : x \rightarrow 5x + 1$ in the domain $-4 \leq x \leq 2$

What are the domain and range of each graph?

What are the minimum and maximum values of each graph?

Definitions

Function A function is a relation (rule) that assigns each element in the domain to exactly one element in the range.

Domain The set of all the values that may be input into a function. That is, the set of all the values the independent variable may assume. Graphically, the domain is the set of all the *x* co-ordinates.

Range The set of all the values that are output when the function is evaluated at all the input values from the domain. That is, the set of all the values the dependent variable may assume. Graphically, the range is the set of all the *y* co-ordinates.

Graphing Quadratic Functions

A quadratic function is usually given in the form $f : x \rightarrow ax^2 + bx + c$, $a \neq 0$, and *a*, *b*, *c* are constants. To draw a quadratic function a table is drawn using the given values of *x* to find the corresponding values of *y*. These points are plotted and joined by a smooth curve.

1. Work out each column separately, i.e. all the x^2 terms first, then all the *x* terms and finally the constant term (watch for patterns in the numbers).
2. Work out each corresponding value of *y*.
3. The **only** column that changes sign is the *x* term (middle) column. If the given values of *x* contain 0, then the x term column will make one sign changes, either from + to – or from – to +, where $x = 0$.
4. The other two columns **never** change sign. They remain either all +'s or all –'s. These columns keep the sign given in the question.

Note: Decide where to draw the *X* and *Y* axes by looking at the table to see what the largest and smallest values of *x* and *y* is. In general, the units on the *X* axis are larger than the units on the *Y* axis. Try to make sure that the graph extends almost the whole width and length of the page.

Quadratic Functions

Example

Graph the quadratic function

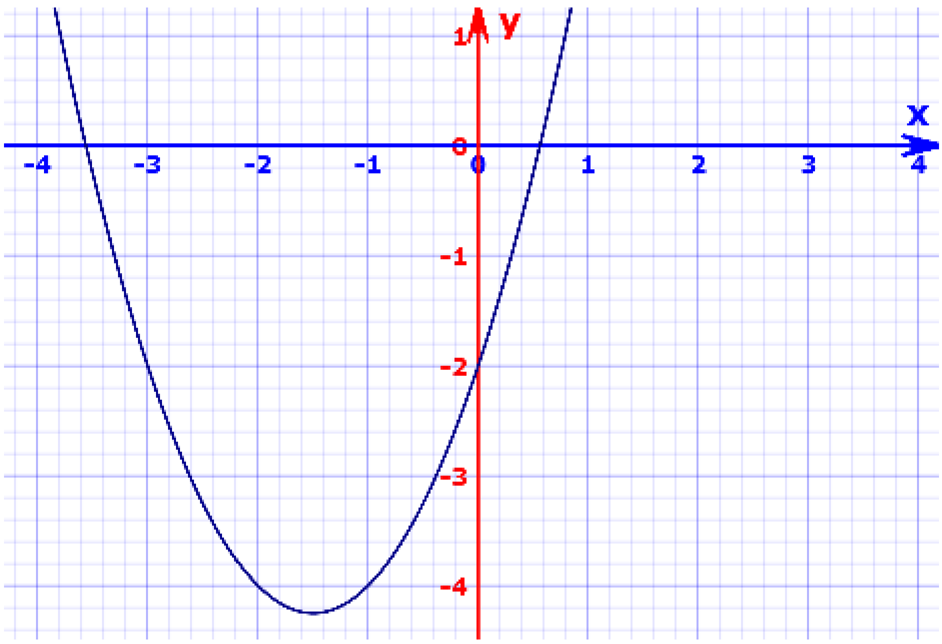
$f : x \rightarrow x^2 + 3x - 2$, in the domain $-5 \leq x \leq 2$, $x \in \mathbf{R}$

Solution

A table is drawn with the given values of *x*, from –5 to 4, to find the corresponding values of *y*.

Let $y = f(x) \Rightarrow y = x^2 + 3x - 2$

x	$x^2 + 3x - 2$	y
-5	25 – 15 – 2	8
-4	16 – 12 – 2	2
-3	9 – 9 – 2	-2
-2	4 – 6 – 2	-4
-1	1 – 3 – 2	-4
0	0 + 0 – 2	-2
1	1 + 3 – 2	2
2	4 + 6 – 2	8



Questions

Graph each of the following functions in the given domain ($x \in \mathbf{R}$ in each case):

$f : x \rightarrow x^2 - 3x + 2$ in the domain $-1 \leq x \leq 4$.

$f : x \rightarrow x^2 - 2x - 3$ in the domain $-2 \leq x \leq 4$.

$f : x \rightarrow x^2 + 2x - 8$ in the domain $-5 \leq x \leq 3$.

$g : x \rightarrow 2x^2 - 3x - 8$ in the domain $-3 \leq x \leq 4$.

$h : x \rightarrow 2x^2 - x - 3$ in the domain $-2 \leq x \leq 3$.

What are the domain and range of each graph?

What are the minimum and maximum values of each graph?

Section 11D: Graphs of Quadratic Functions

The following example will help revise the steps in drawing the graph of a quadratic function.

Example 1
Draw the graph of the function $y = x^2 - 2x - 3$ in the domain $-2 \leq x \leq 4$.
Use your graph to find

(i) the values of x for which $y = 0$

(ii) the values of x for which $y = 2$

(iii) the minimum point of the curve

(iv) the equation of the axis of symmetry of the curve

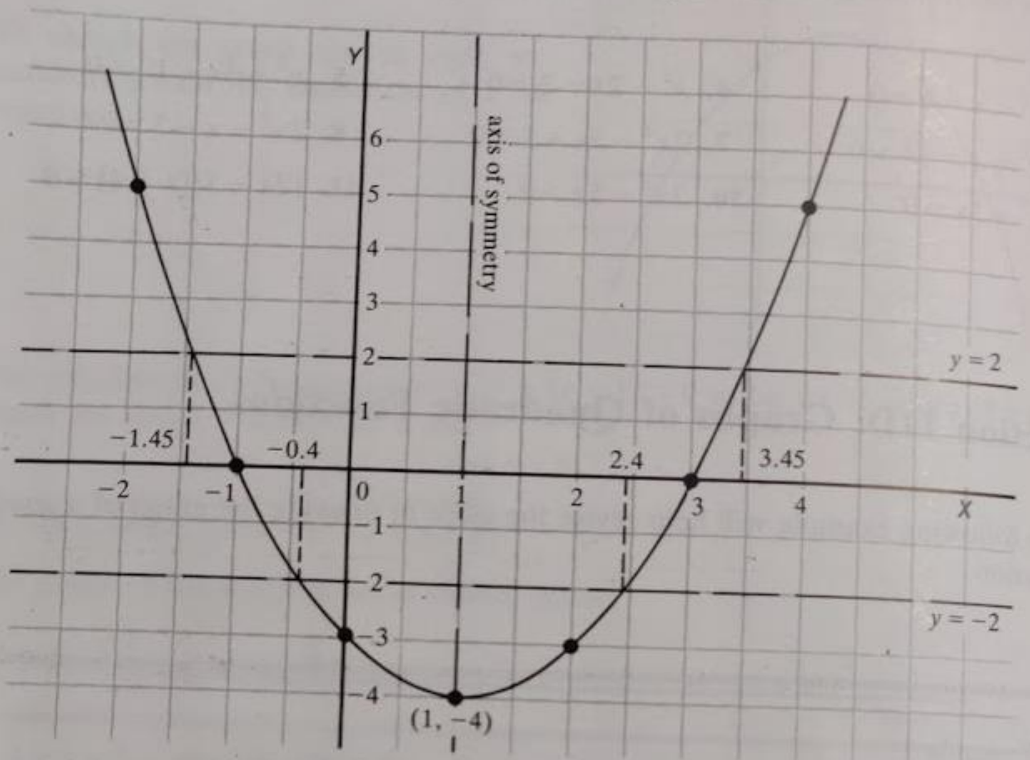
(v) the roots of the equation $x^2 - 2x - 1 = 0$

We set out a table of ordered pairs as follows:

$x =$	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
$y =$	5	0	-3	-4	-3	0	5

The ordered pairs are $(-2, 5)$, $(-1, 0)$, $(0, -3)$, $(1, -4)$, $(2, -3)$, $(3, 0)$ and $(4, 5)$.

Plotting these ordered pairs we get the following curve:



(i) The values of x for which $y = 0$ are $x = -1$ and $x = 3$.
(These are the x-values of the points on the curve where the y-values are equal to 0, i.e. $(-1, 0)$ and $(3, 0)$).

(ii) To find the values of x for which $y = 2$, draw the line $y = 2$, as shown, and then write down the x-values of the points where this line intersects the curve.

$$\Rightarrow x = -1{\cdot}45 \text{ or } x = 3{\cdot}45$$

(iii) The minimum or lowest point on the curve is $(1, -4)$.

(iv) The equation of the axis of symmetry of the curve is $x = 1$.

(v) To find the roots of the equation $x^2 - 2x - 1 = 0$ from the graph we change the equation so that it is of the form:

$$\begin{aligned} &\textit{graphed function} = k. \\ &\text{i.e. } x^2 - 2x - 1 = 0 \\ &\Rightarrow x^2 - 2x - 3 + 2 = 0 \\ &x^2 - 2x - 3 = -2 \\ &\text{i.e. } y = -2 \end{aligned}$$

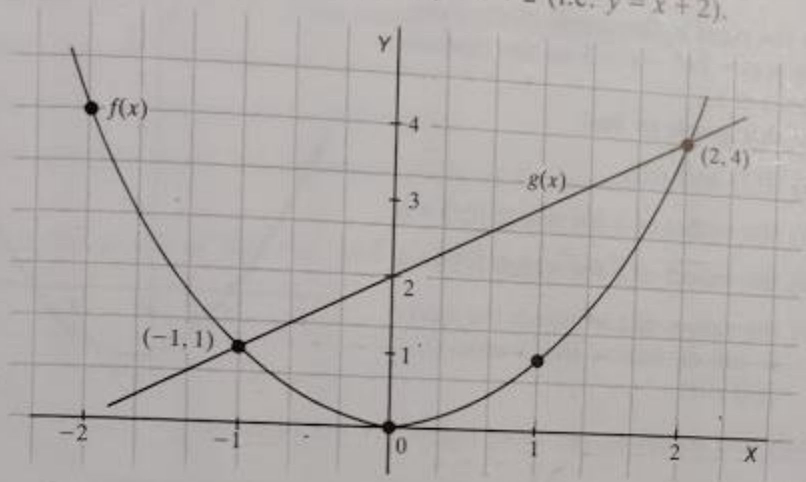
To solve this equation we draw the line $y = -2$ and then find the x-values of the points where this line intersects the curve as shown.

$\Rightarrow x = -0{\cdot}4$ or $x = 2{\cdot}4$.

Intersecting Graphs Involving Quadratics

The diagram below shows the graphs of the functions

$$f(x) = x^2 \text{ (i.e. } y = x^2\text{) and } g(x) = x + 2 \text{ (i.e. } y = x + 2\text{).}$$



Notice that the curve $f(x)$ and the line $g(x)$ intersect at the points $(-1, 1)$ and $(2, 4)$. At these points of intersection $f(x) = g(x)$

i.e. $x^2 = x + 2$

Notice also that the equation $x^2 = x + 2$, i.e., $x^2 - x - 2 = 0$, has as roots the values of x at the points of intersection of the two graphs, i.e., -1 and 2.

Remember

> If $f(x)$ and $g(x)$ are two functions, then the equation $f(x) = g(x)$ can be solved by drawing the graphs of the functions using the same axes and same scales and then writing down the x-values of the points of intersection of the graphs.

Notice also from the graph that $g(x) > f(x)$, i.e. the line is above the curve, from $x = -1$ to $x = 2$

i.e. $g(x) > f(x)$ for $-1 < x < 2$

Test Questions 11D

1. Draw the graph of the function $y = x^2 - 4x$ in the domain $-1 \leq x \leq 5$, $x \in R$.
   Use your graph to find
   (i) the coordinates of the points at which the graph crosses the x-axis
   (ii) the values of x for which $y = 3$
   (iii) the coordinates of the minimum point of the curve
   (iv) the equation of the axis of symmetry of the curve.

2. Draw the graph of the function $f(x) = x^2 + x - 6$ in the domain $-4 \leq x \leq 3$. Use your graph to solve these equations:

(i) $f(x) = 0$ (ii) $f(x) = 4$ (iii) $x^2 + x - 2 = 0$

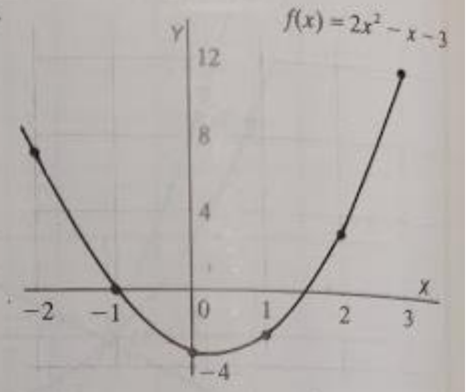
3. On the right is the graph of the function $f(x) = 2x^2 - x - 3$ in the domain $-2 \leq x \leq 3$.
Use this graph to find

(i) $f(3)$ and $f(0)$

(ii) the values of x for which $f(x) = 0$

(iii) the values of x for which $f(x) = 6$

(iv) the values of x at which the curve is on or below the x-axis (i.e. $f(x) \leq 0$)



4. Copy and complete the table below for the function $y = 2 + x - x^2$.

$x =$	−2	−1	0	1	2	3	4
$2 =$	2		2			2	
$x =$	−2		0			3	
$-x^2 =$	−4		0			−9	
$y =$	−4		2			−4	

(i) Draw a graph of the function in the domain $-2 \leq x \leq 4$.

(ii) Use your graph to solve these equations

(a) $2 + x - x^2 = 0$ (b) $2 + x - x^2 = -3$.

(iii) Write down the coordinates of the maximum point of the curve.

(iv) For what values of x in the given domain is the graph on or below the line $y = 0$ (i.e. the x-axis)?

5. Copy and complete the following table for the function $f(x) = 2x^2 + 3x - 6$, $x \in R$.

$x =$	−4	−3	−2	−1	0	1	2	3
$2x^2 =$			8				8	
$3x =$			−6				6	
$-6 =$			−6				−6	
$f(x) =$			−4				8	

Hence draw a graph of the function $f(x) = 2x^2 + 3x - 6$ in the domain $-4 \leq x \leq 3$.

Use your graph to find

(i) the roots of the equation $f(x) = 0$.

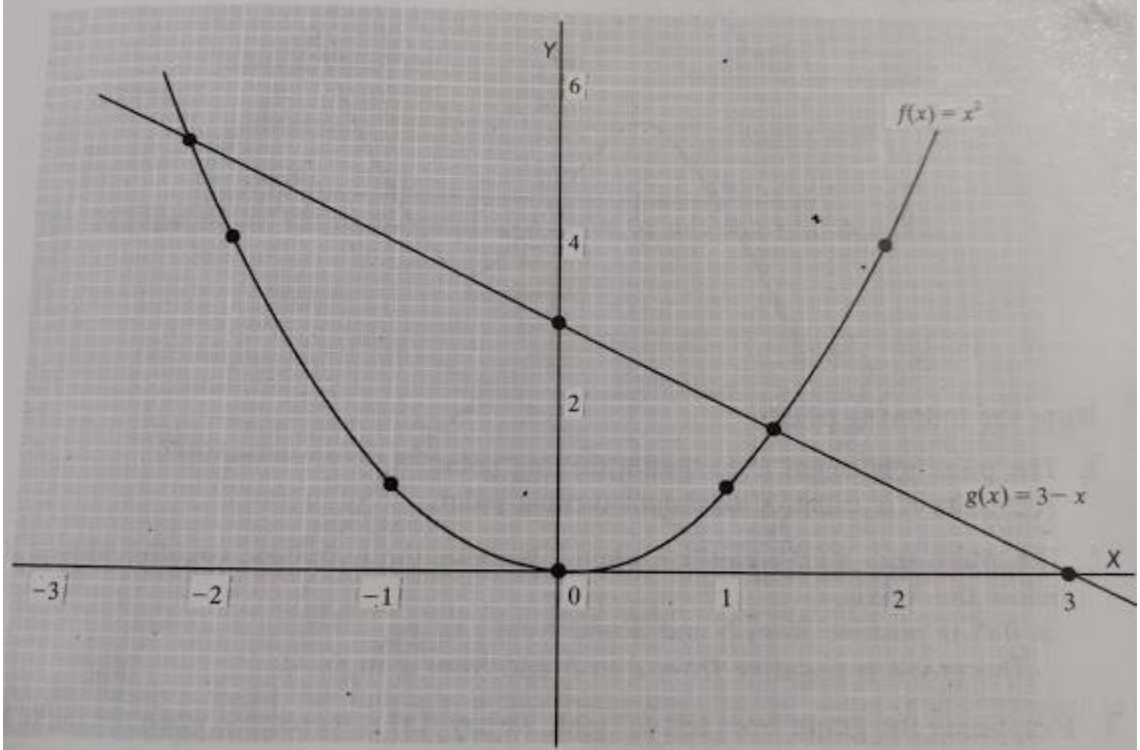
(ii) the values of x in the given domain for which $f(x) > 0$.

(iii) the roots of the equation $2x^2 + 3x - 6 = 1$.

(iv) the roots of the equation $f(x) + 2 = 0$.

(v) the coordinates of the minimum point of the curve.

6. Below is the graph of $f(x) = x^2$ and $g(x) = 3 - x$.



Use the graph to find

(i) the values of x for which $f(x) = g(x)$

(ii) the range of values for x in the domain $-3 \leq x \leq 2$ for which $f(x) \leq g(x)$

(iii) the range of values of x for which $f(x) \leq 1$

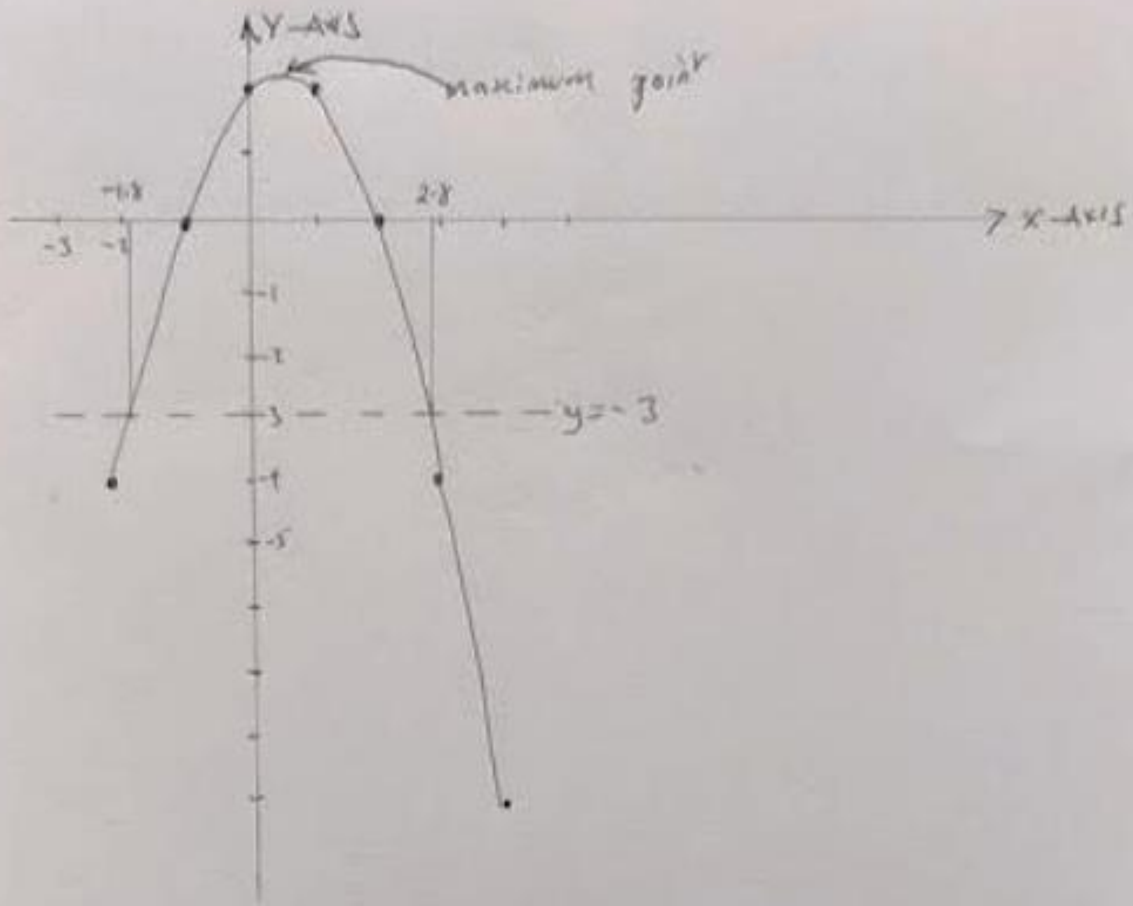
(iv) the value of $f(x) + g(x)$ when $x = -1$

(v) the value of $f(x) - g(x)$ when $x = 0$

Q4 Solution

4 (i)

x	-2	-1	0	1	2	3	4	$\frac{1}{2}$
2	2	2	2	2	2	2	2	2
x	-2	-1	0	1	2	3	4	$\frac{1}{2}$
$-x^2$	-4	-1	0	-1	-4	-9	-16	$-\frac{1}{4}$
$f(x)$	-4	0	2	2	0	-4	-10	$2\frac{1}{4}$



(ii) (a) Solve $2 + x - x^2 = 0$

$x = -1$ and $x = 2$

(b) $2 + x - x^2 = -3$

$f(x) = -3$ $(y = -3)$

$x = -1.8$ and $x = 2.8$

(iii) $x =$

Maximum point $(\frac{1}{2}, 2\frac{1}{4})$

(iv) $\{x \leq -1\} \cup \{x \geq 2\}$

In the given domain, answer $= \{-2 \leq x \leq -1\}$ and $\{2 \leq x \leq 4\}$

Q5 Solution

Q5 $f(x) = 2x^2 + 3x - 6, \quad x \in R \quad \{-4 \leq x \leq 3\}$

x	-4	-3	-2	-1	0	1	2	3	4
$2x^2$	32	18	8	2	0	2	8	18	32
$3x$	-12	-9	-6	-3	0	3	6	9	12
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
$f(x)$	14	3	-4	-7	-6	-1	8	21	38

(i) Roots of $f(x) = 0$

$x = -2.65$ and $x = 1.15$

(ii) x-values for $f(x) > 0$ $\qquad f(x) > 0$ (positive)

$\{-4 \leq x \leq -2.65\}$ and $\{1.15 \leq x \leq 3\}$

(iii) Roots of $2x^2 + 3x - 6 = 1$

$f(x) = 1$

$x = -2.75$ and $x = 1.25$

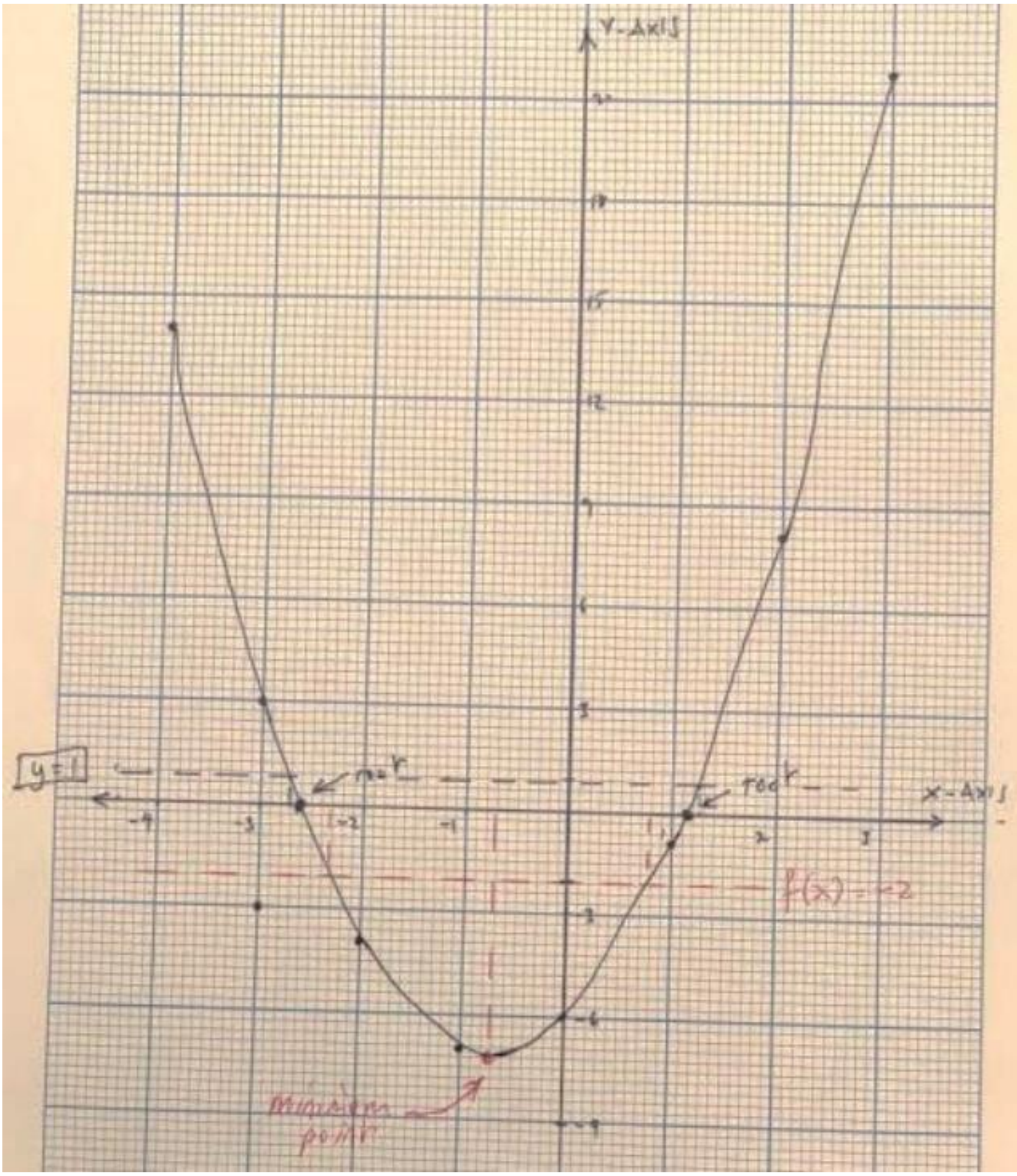
(iv) Roots of $f(x) + 2 = 0$

$f(x) = -2$

$x = -2.35$ and $x = 0.85$

(v) Coordinates of minimum point

$(-0.75, -7.125)$

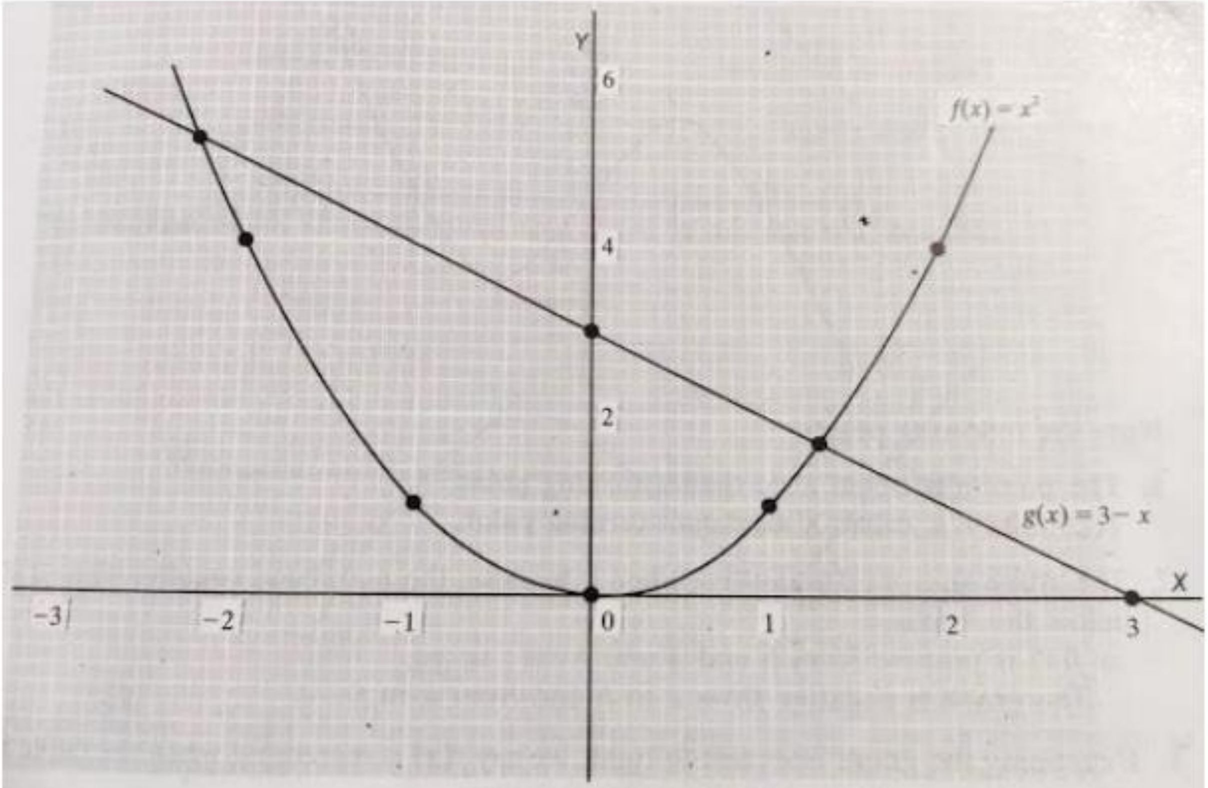


Y-AXIS
X-AXIS
y=1
root
root
f(x) = -2
minimum point

Q6 Question & Solution

Q6

$f(x) = x^2$ $g(x) = 3 - x$



Use the graph to find

(i) the values of x for which $f(x) = g(x)$

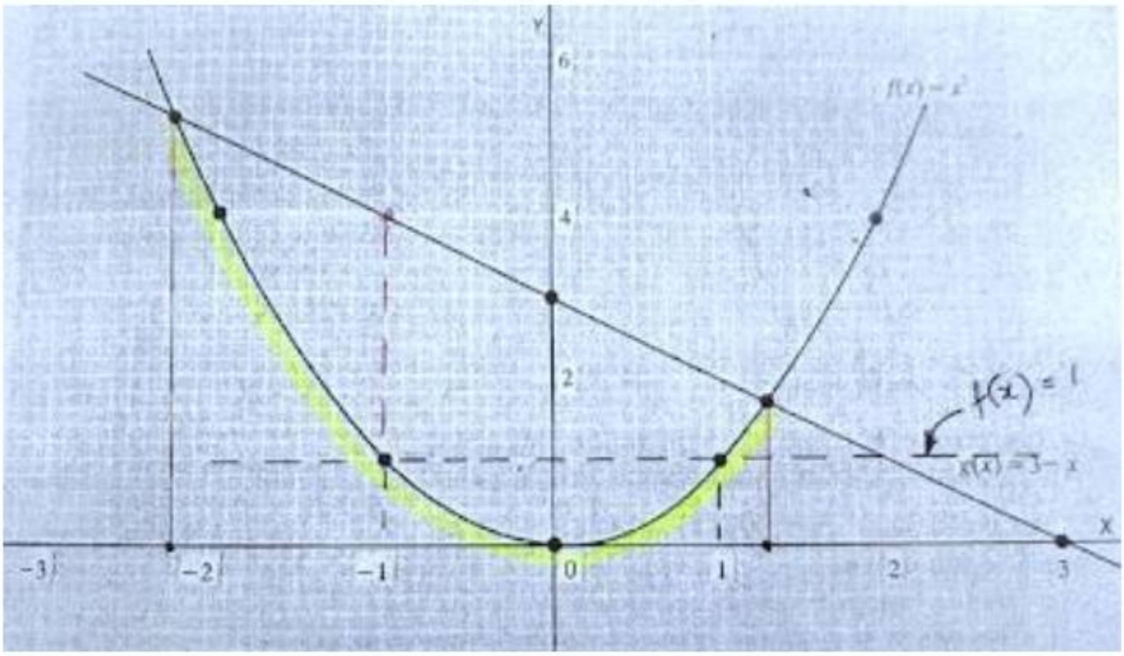
(ii) the range of values for x in the domain $-3 \leq x \leq 2$ for which $f(x) \leq g(x)$

(iii) the range of values of x for which $f(x) \leq 1$

(iv) the value of $f(x) + g(x)$ when $x = -1$

(v) the value of $f(x) - g(x)$ when $x = 0$

Solutions



i. -2.3, 1.3

ii. -2.3≤x≤1.3

iii. -1≤x≤1

iv. $f(x) = 1, g(x) = 4\ therefore\ f(x) + g(x) = 1 + 4 = 5$

v. $f(x) = 0, g(x) = 3\ therefore\ f(x) - g(x) = 0 - 3 = -3$

Draw a graph in a given domain

1	Draw the graph of the following $f(x) = 6x^2 + x - 2$ in the domain -2 ≤ x ≤ 2 Use your graph to solve $6x^2 + x - 2 = 0$ What are the coordinates of the minimum point? What is the range of x values for $f(x) < 0$
2	Draw the graph of the following $f(x) = 3x^2 + 11x - 20$ in the domain -6 ≤ x ≤ 3 Use your graph to solve $3x^2 + 11x - 20 = 0$ What are the coordinates of the minimum point? What is the range of x values for $f(x)$ is positive? What is the rang of x values where f(x) is decreasing?

Pre – Assignment (Important)

Q4 (a) f and g are functions such that

$$f: x \to 2x + 7 \quad and \quad g: x \to 3x - 2$$

(i) Find $f(-1),\ f(0),\ g\left(\frac{1}{3}\right),\ g(-4)$

(ii) The value of x when $f(x) = -5$

(iii) The value of x when $f^{-1}(x) = 10$

(iv) For what value of x is $f(x) = g(x)$

(v) Evaluate $fg(x) + gf(x) - 1$

(b) Using the same axes and the same scales, graph the functions

$$f: x \to x^2 + x - 3$$
$$g: x \to 2x + 1$$

in the domain $-4 \leq x \leq 3$

Use the graph to find, as accurately as you can

(i) The roots of the equation $x^2 + x - 3 = 0$

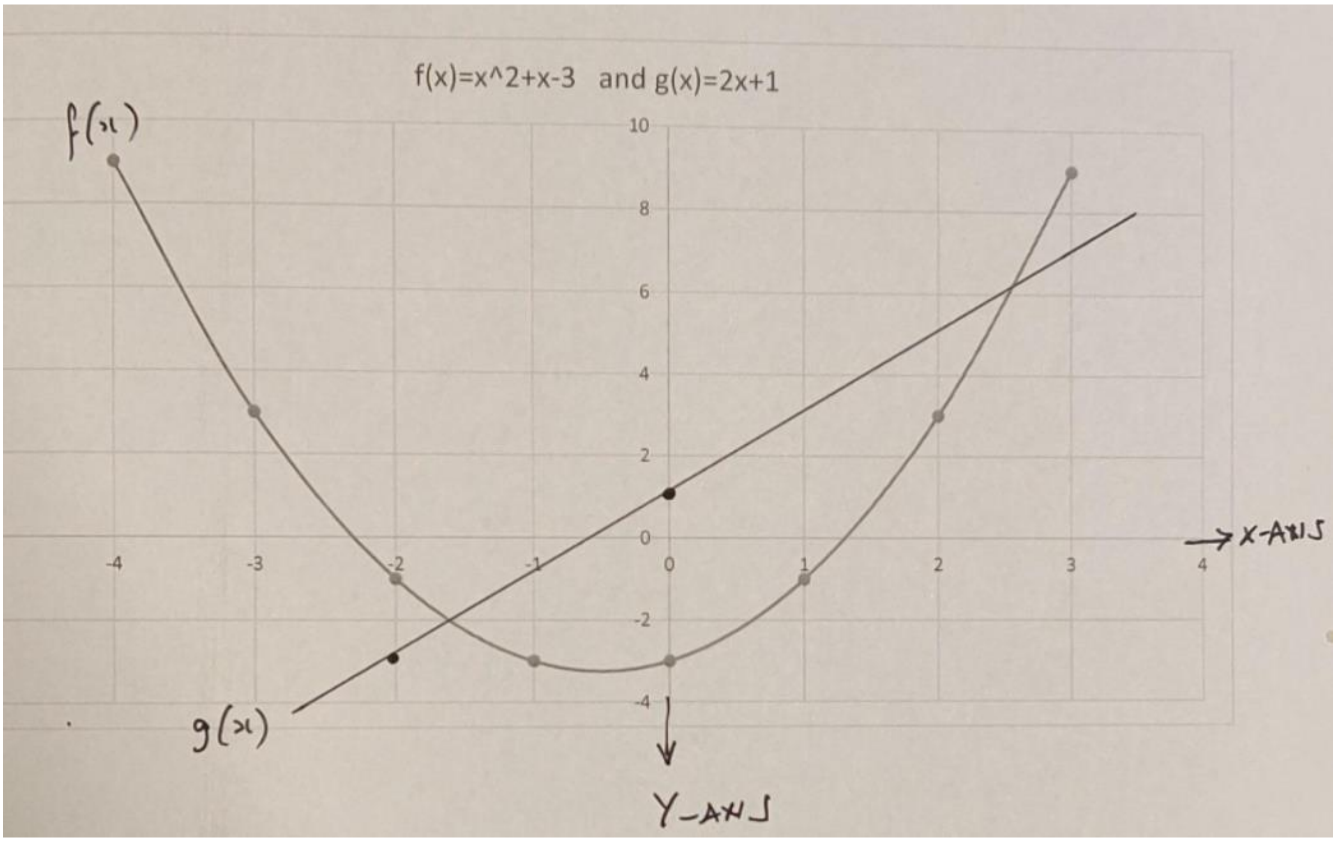
(ii) The roots of the equation $x^2 + x - 1 = 0$

(iii) The minimum value of $f(x)$

(iv) The range of x values for which $f(x) < g(x)$

(v) The values of x for which $f(x) = g(x)$

(vi) The values of x for which $f(x)$ is increasing



Max/min points – increasing/decreasing

Functions and Graphs Exercises – Vertex Max Min

Draw the graph of $f(x) = x^2 + x + 1$ in the domain $\{-4 \le x \le 2\}$

Use your graph

- to estimate the values of x when $y = 5$.
- the coordinated of the minimum point
- the range of x values for $f(x)>0$
- the range of x values for f(x) increasing
- to find the axis of symmetry

Draw the graph of $x^2 + y^2 = 4$ and $x + y = 1$ on the same axis. Domain = $\{-2 \le x \le 2\}$

Use your graph to

- solve $x^2 + y^2 = 4$ and $x + y = 1$

Draw the graph of $f(x) = x^2+2x+1$ in the domain $\{-5 \le x \le 5\}$

Use your graph to

- coordinates of vertex
- axis of symmetry
- x-intercept
- y-intercept
- coordinates of max/min
- value
- domain
- range

Draw the graph of $f(x) = 3x^2-6x+4$ in the domain $\{-2 \le x \le 4\}$

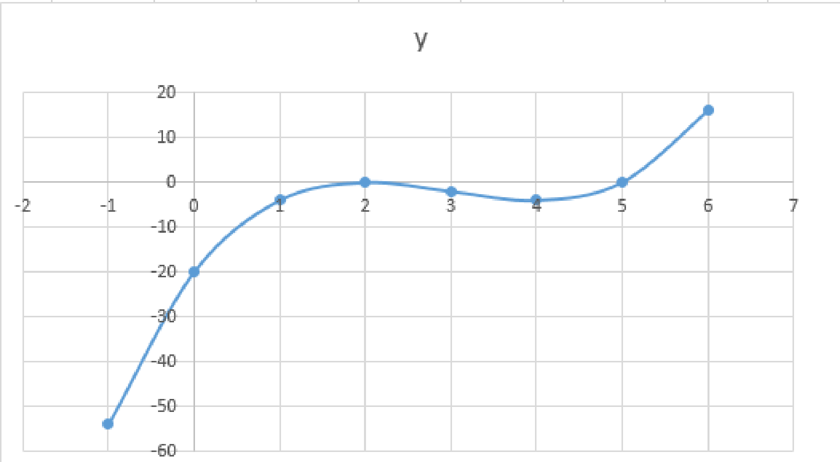
Use your graph to

- coordinates of vertex
- axis of symmetry
- x-intercept

- y-intercept
- coordinates of max/min
- value
- domain
- range

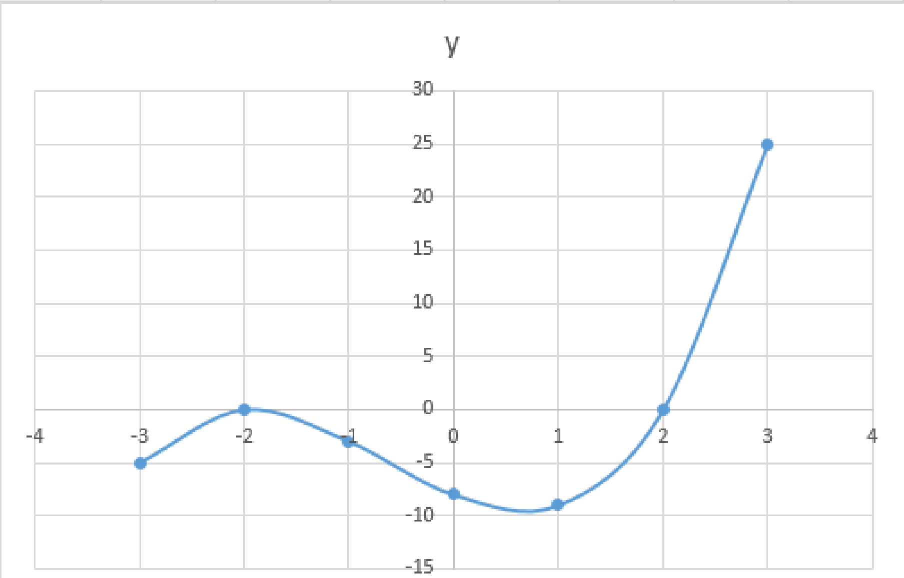
Draw a graph of $f(x) = x^3 -9x^2 +24x -20$

	A	B	C	D	E	F	G	H	I
1	f(x)=x^3-9x^2+24x-20								
2									
3	x	-1	0	1	2	3	4	5	6
4	x^3	-1	0	1	8	27	64	125	216
5	-9^2	-9	0	-9	-36	-81	-144	-225	-324
6	+24x	-24	0	24	48	72	96	120	144
7	-20	-20	-20	-20	-20	-20	-20	-20	-20
8	y	-54	-20	-4	0	-2	-4	0	16



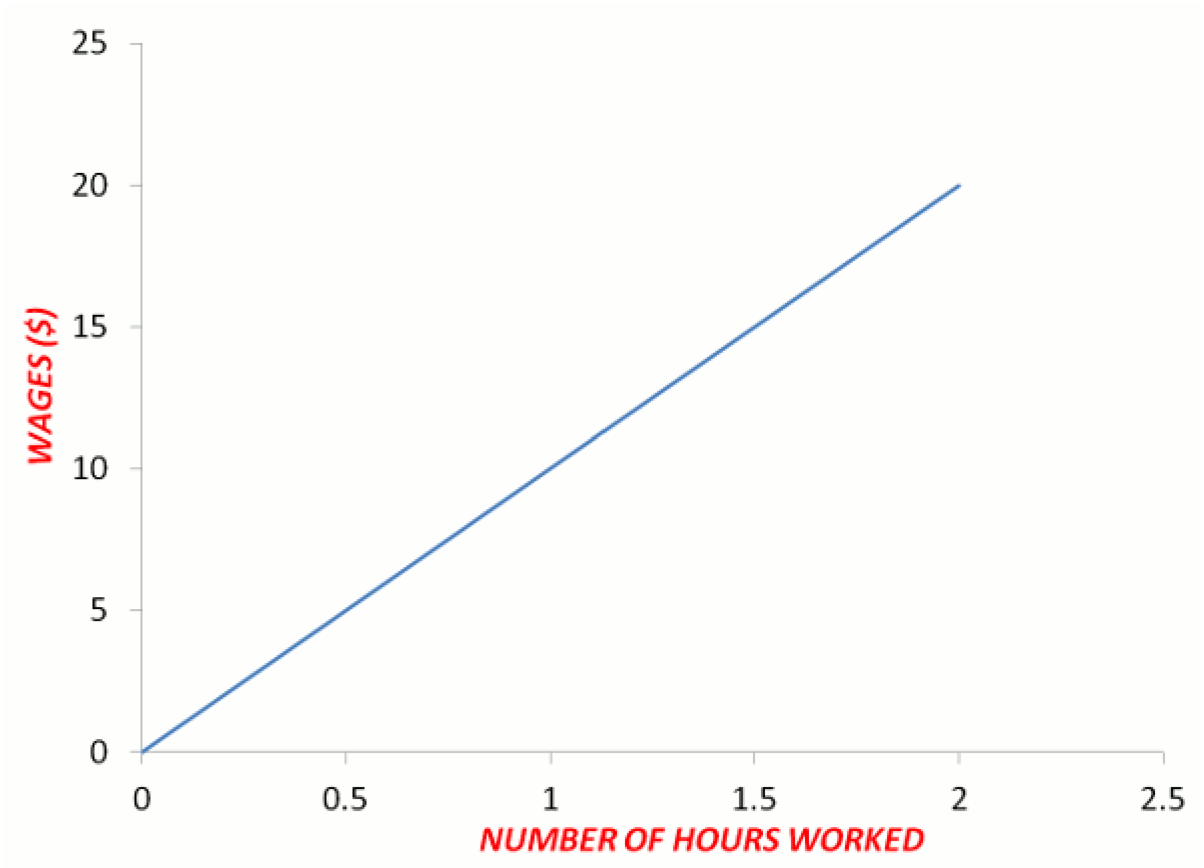
Draw a graph of $f(x) = x^3 +2x^2 -4x -8$

	A	B	C	D	E	F	G	H
1	f(x)=x^3+2x^2-4x-8							
2								
3	x	-3	-2	-1	0	1	2	3
4	x^3	-27	-8	-1	0	1	8	27
5	2x^2	18	8	2	0	2	8	18
6	-4x	12	8	4	0	-4	-8	-12
7	-8	-8	-8	-8	-8	-8	-8	-8
8	y	-5	0	-3	-8	-9	0	25

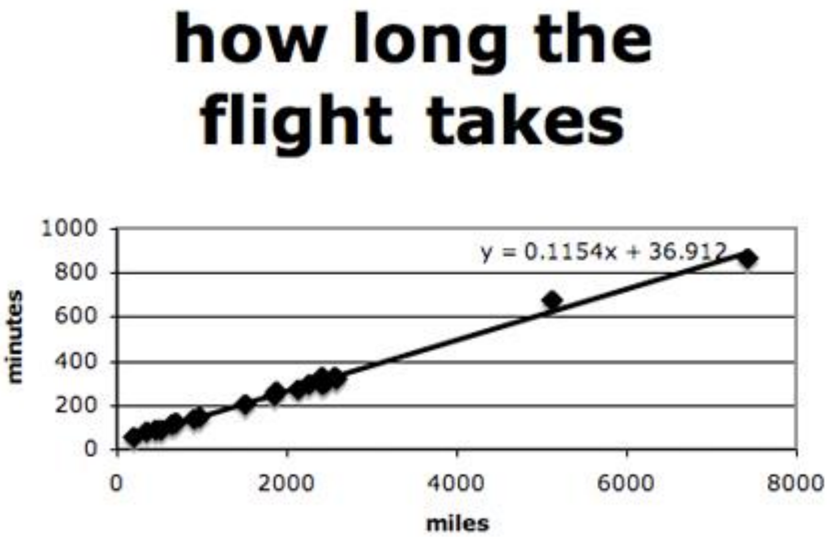


Linear Functions and Graphs

Example 1

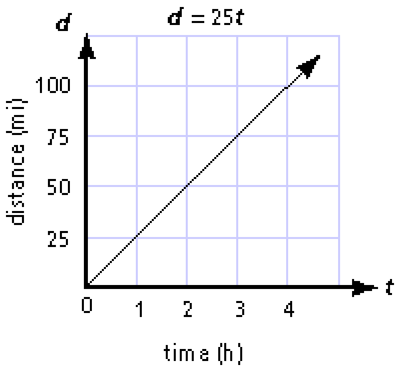


Example 2



Example 3

Graph of the relationship between distance and time when rate is a constant 25 miles per hour.



Example 4