Cork Education and Training Board

Programme Module for

MATHEMATICS

leading to Level 5 QQI

MATHEMATICS [5N1833]

Introduction

This programme module may be delivered as a standalone module leading to certification in a QQI minor award. It may also be delivered as part of an overall validated programme leading to a Level 5 QQI Certificate.

The teacher/tutor should familiarise themselves with the information contained in **Cork Education and Training Board's** programme descriptor for the relevant validated programme prior to delivering this programme module.
The programme module is structured as follows:

1. Title of Programme Module
2. QQI Component Title and Code
3. Duration in hours
4. Credit Value of QQI Component
5. Status
6. Special Requirements
7. Aim of the Programme Module
8. Objectives of the Programme Module
9. Learning Outcomes
10. Indicative Content
11. Assessment a. Assessment Technique(s) b. Mapping of Learning Outcomes to Assessment Technique(s) c. Guidelines for Assessment Activities
12. Grading
13. Learner Marking Sheet(s), including Assessment Criteria

Integrated Delivery and Assessment

The teacher/tutor is encouraged to integrate the delivery of content where an overlap between content of this programme module and one or more other programme modules is identified. This programme module will facilitate the learner to develop the academic and vocational language, literacy and numeracy skills relevant to the themes and content of the module.

Likewise the teacher/tutor is encouraged to integrate assessment where there is an opportunity to facilitate a learner to produce one piece of assessment evidence which demonstrates the learning outcomes from more than one programme module. The integration of the delivery and assessment of level 5 Communications and level 5 Mathematics modules with that of other level 5 modules is specifically encouraged, as appropriate.

Indicative Content

The indicative content in Section 10 does not cover all teaching possibilities. The teacher/tutor is encouraged to be creative in devising and implementing other approaches, as appropriate. The use of examples is there to provide suggestions. The teacher/tutor is free to use other examples, as appropriate. The indicative content ensures all learning outcomes are addressed but it may not follow the same sequence as that in which the learning outcomes are listed in Section 9. It is the teacher's/tutor's responsibility to ensure that all learning outcomes are included in the delivery of this programme module.

1. Title of Programme Module

Mathematics

2. Component Name and Code

Mathematics [5N1833]

3. Duration in Hours

150 hours (Typical learner effort, to include both directed and self-directed learning.)

4. Credit Value

15 Credits

5. Status

This programme module maybe compulsory or optional within the context of the validated programme. Please refer to the relevant programme descriptor, section 9 Programme Structure.

6. Special Requirements

None. However the following are required by all students taking this module:

1. A set of formulae and tables at examination
2. Calculators to be available to each candidate at examination

7. Aim of the Programme Module

The programme module aims to equip the learner with the knowledge, skill and competence to apply a broad range of mathematical skills and tools to a wide variety of contexts, with some theoretical understanding.

8. Objectives of the Programme Module

- To facilitate the learner in their progression of using mathematics for work and life
- To assist the learner to develop the academic and vocational language, literacy and numeracy skills related to Mathematics[5N1833] through the medium of the indicative content
- To enable the learner to take responsibility for his her own learning
- To assist the learner to apply mathematics in a variety of real life situations
- To enable the learner to acquire mathematical skill in problem solving and mathematical investigations in the following areas: mathematical modelling, statistics and probability, graphing functions, differential and integral calculus, complex numbers and trigonometry
- To facilitate the learner to develop competence in problem solving, mathematical, computation, mathematical thinking and conceptual development

9. Learning Outcomes of Level 5: Mathematicss [5N1833]

Learners will be able to:

1 MODELLING USING MATHEMATICS

1.1 Explain the concept of a mathematical model to include the difference between mathematical models and physical models

1.2 Explain the modeling process in diagrammatic form

1.3 Solve simple mathematical models to include identifying situations requiring mathematical modeling, and using appropriate mathematical skills and processes

1.4 Apply simple mathematical models to explain and predict behavior

2 STATISTICS AND PROBABILITY

2.1 Discuss statistical concepts to include discrete and continuous variables, sampling, variance, skewness

2.2 Present information in a range of graphical and tabular forms, using pie charts, trend graphs, correlation diagrams (+/-), cumulative frequency curves, histograms and frequency tables with both discrete and continuous variables

2.3 Calculate the statistics for measuring and contrasting averages and dispersion of grouped data by calculating the mean, mode, median, weighted average, range, inter-quartile range and standard deviation

2.4 Calculate the number of possible outcomes on tests with no repetitions by using the Fundamental Principle of Counting, and Permutations and Combinations

2.5 Demonstrate an understanding of relative frequency and probability by using Information Technology simulations

2.6 Solve simple probability problems of one or two events including where two events are mutually exclusive and where two events are independent

2.7 Discuss findings, to include interpretation of results and distortions which may arise, and reasons for findings

3 FUNCTIONS AND GRAPHS

3.1 Describe the properties of basic mathematical functions to include linear, quadratic, exponential, log and trigonometric functions

3.2 Define the inverse of a function

3.3 Graph linear and quadratic functions showing the relationship between the domain and range

3.4 Derive the inverse of a function from its algebraic expression

3.5 Calculate the equation of a straight line using a range of formulae to include distance between two points, slope, parallel lines and perpendicular lines

3.6 Solve maximum and minimum problems with limitations given by linear inequalities from graphs of linear inequalities and half planes

3.7 Analyse graphs of linear and quadratic functions for important properties to include domain and range, maximum and minimum values, increasing and decreasing intervals, periodicity

4 CALCULUS

4.1 Outline the key concepts of calculus to include limits, differentiation and integration

4.2 Explain the fundamental theorem of calculus

4.3 Calculate average rates of change for related variables *x* and *y* for a variety of standard functions *y=f(x)*

4.4 Differentiate simple standard functions using a table of derivatives

4.5 Use the Product Rule, Quotient Rule and Chain Rule to calculate the derivative of composite functions

4.6 Integrate standard integrals, polynomials, trigonometric and exponential functions

4.7 Calculate the area enclosed between a curve and the *x*-axis using integration

4.8 Apply differentiation to solve simple rates of change models to include maximum and minimum

4.9 Apply integration to solve simple practical real life problems

5 COMPLEX NUMBERS

5.1 Explain what is meant by a complex number

5.2 Represent complex numbers on the Argand diagram to include distinguishing between the modulus and the argument

5.3 Solve quadratic equations with complex roots

5.4 Perform mathematical functions on complex numbers including addition, subtraction, multiplication, division, conjugate, modulus, and plot on an Argand diagram

5.5 Apply de Moivre's Theorem to finding powers of *Z* and the cube root of 1

6 TRIGONOMETRY

6.1 Explore the uses of trigonometry in everyday life.

6.2 Define sine, cosine and tangent functions as related to the unit circle

6.3 Solve practical, simple problems using appropriate trigonometric formulae and terminology, including the sine, cosine and tangent ratios for right angled triangles, *area of triangle=1/2absin C*, Sine Rule, Cosine Rule

6.4 Analyse the functions *y = sinx, y = cosx, y = tanx and y = asinbx* from plotted graphs by determining period, and amplitude.

10. Indicative Content

This section provides suggestions for programme content but is not intended to be prescriptive. The programme module can be delivered through classroom based learning activities, group discussions, one-to-one tutorials, field trips, case studies, role play and other suitable activities, as appropriate.

Section 1: MODELLING USING MATHEMATICS

The learner will be facilitated to:

1.1 Explain the concept of a mathematical model to include the difference between mathematical models and physical models

- Examine real life examples of mathematical and physical models and explain the difference between them

1.2 Explain the modeling process in diagrammatic form

- Describe complete modeling processes using diagrams

1.3 Solve simple mathematical models to include identifying situations requiring mathematical modeling, and using appropriate mathematical skills and processes.

- Examine linear mathematical models to include linear motion under constant acceleration
- Apply mathematical skills to obtain useful answers to real problems

1.4 Apply simple mathematical models to explain and predict behavior

- Use mathematical models to explain and predict behavior, for example, limitations of materials and labor, and then determining the optimal production levels to maximise profits under those conditions.

Section 2: STATISTICS AND PROBABILITY

The learner will be facilitated to:

2.1 Discuss statistical concepts to include discrete and continuous variables, sampling, variance, skewness

- Distinguish between discrete and continuous data
- Discuss sampling from a population to estimate characteristics of the whole population
- Examining the distribution of a data set by considering the variance and skewness of the data set

2.2 Present information in a range of graphical and tabular forms, using pie charts, trend graphs, correlation diagrams (+/-), cumulative frequency curves, histograms and frequency tables with both discrete and continuous variables

- Display data in tabular form
- Graph discrete data using pie charts and histograms
- Graph continuous data using cumulative frequency curves and trend graphs

2.3 Calculate the statistics for measuring and contrasting averages and dispersion of grouped data by calculating the mean, mode, median, weighted average, range, inter-quartile range and standard deviation

- Determine the mean, mode, median, weighted average, range, inter-quartile range and standard deviation for different data sets

2.4 Calculate the number of possible outcomes on tests with no repetitions by using the Fundamental Principle of Counting and Permutations and Combinations

- Use permutations to determine the number of ways a set of elements can be arranged in a particular order
- Use combinations to determine the number of ways of selecting several elements from a larger set

2.5 Demonstrate an understanding of relative frequency and probability by using Information Technology simulations

- Use ICT resources to demonstrate relative frequencies and probabilities

2.6 Solve simple probability problems of one or two events including where two events are mutually exclusive and where two events are independent

- Define mutually exclusive events and independent events
- Calculate probabilities using the *Addition Law* and *Multiplication Law* of

probability

2.7 Discuss findings, to include interpretation of results and distortions which may arise, and reasons for findings

- interpret the results and findings and any distortions which may arise after applying statistical analysis

Section 3: FUNCTIONS AND GRAPHS

The learner will be facilitated to:

3.1 Describe the properties of basic mathematical functions to include linear, quadratic, exponential, log and trigonometric functions

- graph linear, quadratic, exponential, log and trigonometric functions and describe their properties

3.2 Define the inverse of a function

- describe functions that are the reverse of another function

3.3 Graph linear and quadratic functions showing the relationship between the domain and range

- graph linear and quadratic functions for a given domain and determine the range for the functions

3.4 Derive the inverse of a function from its algebraic expression

- determine the inverse of linear, exponential, log and trigonometric functions

3.5 Calculate the equation of a straight line using a range of formulae to include distance between two points, slope, parallel lines and perpendicular lines

- Calculate the equation of a line given two points, a point and a slope, a point and a parallel line or a point and a perpendicular line

3.6 Solve maximum and minimum problems with limitations given by linear inequalities from graphs of linear inequalities and half planes

- Solve linear programming problems, for example, taking the limitations of materials and labor, and then determining the optimal production levels to maximise profits under those conditions.

3.7 Analyse graphs of linear and quadratic functions for important properties to include domain and range, maximum and minimum values, increasing and decreasing intervals, periodicity

- Determine the maximum and minimum turning points, the domain, the range and the period and intervals of increasing and decreasing for linear and quadratic functions

Section 4: CALCULUS

The learner will be facilitated to:

4.1 Outline the key concepts of calculus to include limits, differentiation and integration

- Introduce the concept of a derivative, which is the primary tool used to calculate rates of change and slopes of tangents
- Define the derivative of the function using limits
- Introduce integration as the inverse of differentiation

4.2 Explain the fundamental theorem of calculus

- Use integration to define the Fundamental Theorem of Calculus

4.3 Calculate average rates of change for related variables *x* and *y* for a variety of standard functions $y = f(x)$

- Use differentiation to calculate rates of change such as velocity and acceleration

4.4 Differentiate simple standard functions using a table of derivatives

- Differentiate functions given in Formula and Tables approved for use in the state examinations

4.5 Use the Product Rule, Quotient Rule and Chain Rule to calculate the derivative of composite functions

- Differentiate composite functions using the Product Rule, Quotient Rule and the Chain Rule

4.6 Integrate standard integrals, polynomials, trigonometric and exponential functions

- Integrate functions given in Formula and Tables approved for use in the state examinations

4.7 Calculate the area enclosed between a curve and the *x*-axis using integration

- Use integration to calculate the area enclosed between the x-axis and linear and quadratic functions

4.8 Apply differentiation to solve simple rates of change models to include maximum and minimum

- Use differentiation to determine the maximum and minimum turning points of quadratic and cubic functions

4.9 Apply integration to solve simple practical real life problems

- Use integration to find the volume under a curve revolved about the x-axis
- Use integration to calculate the work done by a variable force along the *x*-axis

Section 5: COMPLEX NUMBERS

The learner will be facilitated to:

5.1 Explain what is meant by a complex number

- Explain how to evaluate the square root of a negative number

5.2 Represent complex numbers on the Argand diagram to include distinguishing between the modulus and the argument

- Represent complex numbers on a complex plane and calculate the modulus and argument of complex numbers

5.3 Solve quadratic equations with complex roots

- Use the Quadratic Formula so solve quadratic equations with complex roots

5.4 Perform mathematical functions on complex numbers including addition, subtraction, multiplication, division, conjugate, modulus, and plot on an Argand diagram

- Perform operations on complex numbers to include addition, subtraction, scalar multiplication and multiplication and division of complex numbers

- Represent complex numbers on a complex plane and calculate the conjugate, modulus and argument of complex numbers

5.5 Apply de Moivre's Theorem to finding powers of *Z* and the cube root of 1

- Convert complex numbers to polar form
- Use De Moivre's Theorem to calculate the cube roots of unity

Section 6: TRIGONOMETRY

The learner will be facilitated to:

6.1 Explore the uses of trigonometry in everyday life

- Describe everyday uses of trigonometry such as navigation

6.2 Define sine, cosine and tangent functions as related to the unit circle

- Use the unit circle to define sine, cosine, and tangent functions

6.3 Solve practical, simple problems using appropriate trigonometric formulae and terminology, including the sine, cosine and tangent ratios for right angled triangles, *area of triangle=1/2absin C*, Sine Rule, Cosine Rule

- Use the sine, cosine and tangent ratios for right angled triangles to calculate angles
- Calculate areas, lengths of sides and angles in triangles using the Sine Rule, Cosine Rules and using *area of triangle=1/2absin C*

6.4 Analyse the functions *y = sinx, y = cosx, y = tanx and y = asinbx* from plotted graphs by determining period, and amplitude.

- Determine the range and period of functions of the form *y = sinx, y = cosx, y = tanx and y = asinbx*

11. Assessment

11a. Assessment Techniques

Assignments (2)	60%
Examination – Theory	40%

11b. Mapping of Learning Outcomes to Assessment Techniques

In order to ensure that the learner is facilitated to demonstrate the achievement of all the learning outcomes from the component specification; each learning outcome is mapped to an assessment technique(s). This mapping should not restrict an assessor from taking an integrated approach to assessment.

1.1	Explain the concept of a mathematical model to include the difference between mathematical models and physical models	**Assignment 1**
1.2	Explain the modeling process in diagrammatic form	**Assignment 1**
1.3	Solve simple mathematical models to include identifying situations requiring mathematical modeling, and using appropriate mathematical skills and processes	**Assignment 1**
1.4	Apply simple mathematical models to explain and predict behavior	**Assignment 1**
2.1	Discuss statistical concepts to include discrete and continuous variables, sampling, variance, skewness	**Assignment 1**
2.2	Present information in a range of graphical and tabular forms, using pie charts, trend graphs, correlation diagrams (+/-), cumulative frequency curves, histograms and frequency tables with both discrete and continuous variables	**Assignment 1 / Exam**
2.3	Calculate the statistics for measuring and contrasting averages and dispersion of grouped data by calculating the mean, mode, median, weighted average, range, inter-quartile range and standard deviation	**Assignment 1 / Exam**
2.4	Calculate the number of possible outcomes on tests with no repetitions by using the Fundamental Principle of Counting, and Permutations and Combinations	**Assignment 1 / Exam**
2.5	Demonstrate an understanding of relative frequency and probability by using Information Technology simulations	**Assignment 1**
2.6	Solve simple probability problems of one or two events including where two events are mutually exclusive and where two events are independent	**Assignment 1 / Exam**
2.7	Discuss findings, to include interpretation of results and distortions which may arise, and reasons for findings	**Assignment 1**
3.1	Describe the properties of basic mathematical functions to include linear, quadratic, exponential, log and trigonometric functions	**Assignment 1**

3.2	Define the inverse of a function	**Assignment 1**
3.3	Graph linear and quadratic functions showing the relationship between the domain and range	**Assignment 1 / Exam**
3.4	Derive the inverse of a function from its algebraic expression	**Assignment 1 / Exam**
3.5	Calculate the equation of a straight line using a range of formulae to include distance between two points, slope, parallel lines and perpendicular lines	**Assignment 1 / Exam**
3.6	Solve maximum and minimum problems with limitations given by linear inequalities from graphs of linear inequalities and half planes	**Assignment 1 /**
3.7	Analyse graphs of linear and quadratic functions for important properties to include domain and range, maximum and minimum values, increasing and decreasing intervals, periodicity	**Assignment 1 / Exam**
4.1	Outline the key concepts of calculus to include limits, differentiation and integration	**Assignment 2 /**
4.2	Explain the fundamental theorem of calculus	**Assignment 2**
4.3	Calculate average rates of change for related variables *x* and *y* for a variety of standard functions *y=f(x*)	**Assignment 2**
4.4	Differentiate simple standard functions using a table of derivatives	**Assignment 2 / Exam**
4.5	Use the Product Rule, Quotient Rule and Chain Rule to calculate the derivative of composite functions	**Assignment 2 / Exam**
4.6	Integrate standard integrals, polynomials, trigonometric and exponential functions	**Assignment 2 / Exam**
4.7	Calculate the area enclosed between a curve and the *x*-axis using integration	**Assignment 2 / Exam**
4.8	Apply differentiation to solve simple rates of change models to include maximum and minimum	**Assignment 2 / Exam**
4.9	Apply integration to solve simple practical real life problems	**Assignment 2**
5.1	Explain what is meant by a complex number	**Assignment 2**
5.2	Represent complex numbers on the Argand diagram to include distinguishing between the modulus and the argument	**Assignment 2**

5.3	Solve quadratic equations with complex roots	**Assignment 2 / Exam**
5.4	Perform mathematical functions on complex numbers including addition, subtraction, multiplication, division, conjugate, modulus, and plot on an Argand diagram	**Assignment 2 / Exam**
5.5	Apply de Moivres Theorem to finding powers of *Z* and the cube root of 1	**Assignment 2**
6.1	Explore the uses of trigonometry in everyday life.	**Assignment 2**
6.2	Define sine, cosine and tangent functions as related to the unit circle	**Assignment 2**
6.3	Solve practical, simple problems using appropriate trigonometric formulae and terminology, including the sine, cosine and tangent ratios for right angled triangles, *area of triangle=1/2absin C*, Sine Rule, Cosine Rule	**Assignment 2 / Exam**
6.4	Analyse the functions *y = sinx, y = cosx, y = tanx and y = asinbx* from plotted graphs by determining period, and amplitude.	**Assignment 2**

11c. Guidelines for Assessment Activities

The assessor is required to devise assessment briefs and marking schemes/examination papers and marking schemes and outline solutions for the assignments and examination. In devising the assessment briefs/examination papers, care should be taken to ensure that the learner is given the opportunity to **show evidence of achievement of ALL the learning outcomes**. Assessment briefs may be designed to allow the learner to make use of a wide range of media in presenting assessment evidence, as appropriate. Quality assured procedures must be in place to ensure the reliability of learner evidence.

Assignments (2)	**60%**
5 Weeks for each assignment	
Assignment 1- 30% The brief for the first assignment will cover learning outcomes from Modelling using Mathematics, Probability and Statistics and Functions and Graphs *Learners will be required to answer all 5 questions* Section 1 will assess learners knowledge of Modelling using Mathematics Section 2 will assess learners knowledge of Statistics Section 3 will assess learners knowledge of Probability Section 4 will assess learners knowledge of Functions & Graphs Section 5 will assess learners knowledge of Linear Programming **Assignment 2-30%** The brief for the second assignment will cover learning outcomes from Calculus, Complex numbers and Trigonometry *Learners will be required to answer all 5 questions* Section 1 will assess learners knowledge of Differential Calculus Section 2 will assess learners knowledge of Integral Calculus Section 3 will assess learners knowledge of Complex numbers Section 4 will assess learners knowledge of De Moivre's theorem Section 5 will assess learners knowledge of Trigonometry	

Evidence for these assignments may take the form of written, oral, graphic, audio, visual or digital evidence, or any combination of these. Any audio, video or digital evidence must be provided in a suitable format. All instructions for the learner must be clearly outlined in an assessment brief.

Examination – Theory	**40%**
2 hours	
The examination will be based on a range of learning outcomes from all sections and will be 2 hours in duration. The format of the exam will be as follows: **Section A-20%** 10 short questions covering all sections (4 marks each) Candidates are required to answer all ten questions in this section **Section B-10%** 2 structured questions from Modelling using Mathematics, Probability and Statistics and Functions and Graphs (10 marks each) Candidates are required to **answer 2** questions from this section **Section C-10 %** 2 structured questions from Calculus, Complex numbers and Trigonometry (10 marks each) Candidates are required to **answer 2** questions from this section	

12. Grading

Distinction: 80% - 100%
Merit: 65% - 79%
Pass: 50% - 64%
Unsuccessful: 0% - 49%

At levels 4, 5 and 6 major and minor awards will be graded. The grade achieved for the major award will be determined by the grades achieved in the minor award

ASSESSMENT CRITERIA AND MARKING SCHEME GUIDE *MARKING SHEET 1*	**MATHEMATICS** **5N1833** **ASSIGNMENTS 60%**

Learner Name: _____

Centre: _____

ASSESSMENT CRITERIA		*MAXIMUM MARK*	*LEARNER MARK*
Assessment criteria for both assignments • Effective application of mathematics to real life situations by correctly • formulating problems, modelling problems with appropriate mathematics verifying and interpreting results. • Accurate calculations, correct use of formulae. • Coherent format with appropriate use of mathematical symbols, letters and terminology. Logical progression of thought.			
Assignment 1			
Question 1	Modelling using Mathematics	**12**	
Question 2	Statistics	**12**	
Question 3	Probability	**12**	
Question 4	Functions & Graphs	**12**	
Question 5	Linear Programming	**12**	
	SUBTOTAL 1	**60**	
Assignment 2			
Question 1	Differential Calculus	**12**	
Question 2	Integral Calculus	**12**	
Question 3	Complex Numbers	**12**	
Question 4	De Moivres Theorem	**12**	
Question 5	Trigonometry	**12**	
	SUBTOTAL 2	**60**	
	TOTAL MARKS = (SUBTOTAL 1 + SUBTOTAL 2) ÷ 2	**60**	

Internal Assessor Signature: _____ Date: _____

External Authenticator's Signature: _____ Date: _____

ASSESSMENT CRITERIA ***AND*** ***MARKING SCHEME GUIDE*** ***MARKING SHEET 2***	**MATHEMATICS** **5N1833** **EXAMINATION (THEORY) 40%**

Learner Name: _____

Centre: _____

ASSESSMENT CRITERIA	***MAXIMUM MARK***	***LEARNER MARK***
Section A: Short Answer Questions 10 short answer questions, answer all questions (***4 marks each***) Question No.: _____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
_____	**4**	
SUBTOTAL 1:	**40**	
Section B: *Answer any 2 questions* 2 structured questions from Modelling using Mathematics, Probability and Statistics and Functions and Graphs (***10 marks each***) Question No.: _____	**10**	
Question No.: _____	**10**	
Section C: *Answer any 2 questions* 2 structured questions from Calculus, Complex numbers and Trigonometry (***10 marks each***) Question No.: _____	**10**	
Question No.: _____	**10**	
SUBTOTAL 2:	**40**	
TOTAL MARKS = (Subtotal 1 + Subtotal 2) ÷ 2	**40**	

Internal Assessor Signature: _____ Date: _____

External Authenticator's Signature: _____ Date: _____