

## Tallying

Tallying is a useful way of counting or collecting data as shown below. The data can then be represented in a frequency distribution table or grouped frequency distribution table.

Tallying is a way of adding that uses **groups of five**.

You can record your results on a **tally chart**, like this one:

Colour	Tally	Frequency
Red		13
Blue		9
White		24
Black		12
Other		9

As you can see, using groups of five makes it easier to see the total.

### Investigation example

You are carrying out a survey to determine the number of magazines bought by your classmates in the past year. The possible answers are likely to range from **0** to **100**, so you may draw a tally chart with groupings similar to this one:

Number of magazines	Tally	Frequency
0 - 4		8
5 - 9		5
10 - 14		7
15 - 19		3
20 - 24		9
25 - 29		3
30 - 34		0
35 - 39		5
40 - 44		0
45 - 49		3
more than 49		0

The completed frequency table is known as a **grouped frequency table**.

# How to make a Frequency Distribution Table: Examples

## Example 1

**Tally marks** are often used to make a frequency distribution table. For example, let's say you survey a number of households and find out how many pets they own. The results are 3, 0, 1, 4, 4, 1, 2, 0, 2, 2, 0, 2, 0, 1, 3, 1, 2, 1, 1, 3. Looking at that string of numbers boggles the eye; a frequency distribution table will make the data easier to understand.

## Steps

To make the frequency distribution table, first **write the categories** in one column (number of pets):

Number of Pets (x)	Tally	Frequency (f)
0		
1		
2		
3		
4		

## Exercises

Ex 1	Raw data	1	3	4	1	2	1	6	7
	Ordered data								
	Mode =								
	Median =								
	Mean =								

Ex 2	Raw data	1	0	3	5	0	6	7
	Ordered data							
	Mode =							
	Median =							
	Mean =							

Ex 3	Raw data	8	0	3	3	1	7	4	1	4	4
	Ordered data										
	Mode =										
	Median =										
	Mean =										

Ex 4

Find the mean of the following:  $x$ ,  $x+3$ ,  $x+1$ ,  $x-3$ ,  $x-1$

Ex 5

The mean of the numbers 1,  $x$ , 3, 6, 8 is 7, What is the number represented by  $x$ ?

Ex 6

On a particular day the number of goals scored in 30 football matches was recorded as follows. Input the data in a frequency distribution table and calculate the mean/mode/median.

1	3	4	3	2	1	6	5	1	2	3	2	1	2	4	3	1	4	6	2	5	2	4	6	2	1	6	2	1	5
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## Solutions

Ex 1

<b>Raw data</b>	1	3	4	1	2	1	6	7
<b>Ordered data</b>	1	1	1	2	3	4	6	7
<b>Mode =</b>	1							
<b>Median =</b>	2.5							
<b>Mean =</b>	3.125							

Ex 2

<b>Raw data</b>	1	0	3	5	0	6	7
<b>Ordered data</b>	0	0	1	3	5	6	7
<b>Mode =</b>	0						
<b>Median =</b>	3						
<b>Mean =</b>	3.142857143						

Ex 3

<b>Raw data</b>	8	0	3	3	1	7	4	1	4	4
<b>Ordered data</b>	0	1	1	3	3	4	4	4	7	8
<b>Mode =</b>	4									
<b>Median =</b>	3.5									
<b>Mean =</b>	3.5									

Ex 4

Find the mean of the following:  $x$ ,  $x+3$ ,  $x+1$ ,  $x-3$ ,  $x-1$

$$\frac{x + x + 3 + x + 1 + x - 3 + x - 1}{5} = \frac{5x}{5} = x$$

Ex 5

$$\frac{1 + x + 3 + 6 + 8}{5} = 7$$

$$\frac{x + 18}{5} = 7$$

$$x + 18 = 35$$

$$x = 17$$

Ex 6

x	1	2	3	4	5	6
f	7	8	4	4	3	4

Totals  
30

xf      7   16   12   16   15   24      90

Mean = 3

Mode = 2

Median

= 2.5 (Note:  $30/2 = 15$  so its 15th and 16th numbers divided by two)

## Weighted Mean

### Formula

$$\bar{x}_w = \frac{\sum wx}{\sum w}$$

### Example: Calculating grades

Suppose a class has quizzes, homework and three exams. The scores are weighted as follows:

Quiz	HW	Exam1	Exam2	Final
10%	10%	20%	20%	40%

Suppose your averages are as follows:

Quiz	HW	Exam1	Exam2	Final
80%	89%	79%	84%	87%

How is your grade calculated?

Multiply percentages by respective weights (written as decimals)

$$0.1(80)+0.1(89)+0.2(79)+0.2(84)+0.4(87)$$

= 84.3% final grade

### Example: Price Increase

Type of Meat	Pork	Mutton	Beef	Poultry
% Price Increase	2%	8%	3%	5%

$$\text{Mean Price Increase} = \frac{2+8+3+5}{4} = 4.5\%$$

Type of Meat	Pork	Mutton	Beef	Poultry
% Price Increase	2%	8%	3%	5%
Weight	3	1	4	2

$$\text{Weighted Mean} = \frac{6+8+12+10}{3+1+4+2} = \frac{36}{10} = 3.6\%$$

### Exercises

	Shoes	Hardware	Clothes	Food	Housing	Fuel
% rise in year 1	12	19	23	24	8	15
% rise in year 2	18	20	14	16	4	8
Assigned weights	1	2	4	6	4	3

The above table shows the percentage rise in prices of various items and their assigned weights. Which of the two years had the bigger weighted mean of the % price rises?

Students in an American University have to take three subjects: a major, a minor and a general. Their marks in these are weighted with weights 5, 3, 2 respectively.

Here are the marks of 2 students, Alice and Bernard:

	Major score	Minor score	General score
Alice	55	64	72
Bernard	58	60	70

- Which student had the higher mean score?
- Which student had the higher WEIGHTED mean score?

## Answers

1

% rise in Year 1	12	38	92	144	32	45	18.15
% rise in Year 2	18	40	56	96	16	24	12.5
Year 1 Weighted Mean	18.15						
Year 2 Weighted Mean	12.5						

Answer = Year 1 has bigger weighted mean.

2

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Weights	5	3	2					
	Major	Minor	General					<b>Sums</b>
Alice	55	64	72	275	192	144	=>	<b>611</b>
Bernard	58	60	70	290	180	140	=>	<b>610</b>

### Mean Scores

Alice 63.67

Bernard 62.67

### Weighted Mean Scores

Alice 61.1

Bernard 61

I. Alice has highest mean score

II. Alice has highest weighted mean score.

2. Rewrite each of the following arrays of numbers in order of size and then write down (i) the mode, (ii) the median:
- (a) 1, 3, 4, 1, 2, 1, 6, 7      (b) 1, 0, 3, 5, 0, 6, 7
- (c) 8, 11, 2, 5, 8, 7, 8, 2, 5      (d) 8, 0, 3, 3, 1, 7, 4, 1, 4, 4
3. (i) The mean of 3, 7, 8, 10 and  $x$  is 6. Find  $x$ .  
 (ii) The mean of 1,  $x$ , 3, 6 and 8 is 7. Find  $x$ .
4. The mean of four numbers is 15. If three of the numbers are 12, 10 and 13, find the fourth number.
5. The mean height of 6 men is 1.7 m and the mean height of 4 women is 1.6 m. Find
- (i) the total height of the 6 men  
 (ii) the total height of the 4 women  
 (iii) the mean height of the 6 men and 4 women.
6. The mean of six numbers is 12. When a seventh number is added the mean of the seven numbers is 14. Find the seventh number.
7. The following table gives the number of goals scored in 60 matches on a particular week-end:

Goals scored	1	2	3	4	5	6
No. of matches	14	16	8	8	6	8

- (i) Write down the mode of the distribution  
 (ii) Calculate the mean.
8. The numbers 7, 5, 13, 5, 13, 4, 11,  $x$ ,  $y$  have mean 8 and mode 5. Find the values of  $x$  and  $y$ .
9. The table below shows the number of goals scored in 100 football matches on a particular Saturday.

No. of goals scored	0	1	2	3	4	5
No. of matches	10	25	30	25	10	0

- (i) Write down the modal number of goals scored.  
 (ii) Calculate the mean of the distribution.
10. If the mean of the frequency distribution below is 2, find the value of  $x$ .

Variable	0	2	3	4
Frequency				



### Estimating the Mean from a Grouped Frequency Distribution

When dealing with a large number of variables, such as the ages of people in a certain district, it is often more convenient to arrange the data in **groups** or **classes**. Thus, when recording the ages of people, the results could be grouped (0–9) years, (10–19) years . . . etc.

The **grouped frequency distribution** table below shows the marks (out of 25) achieved by 50 students in a test.

Marks achieved	1–5	6–10	11–15	16–20	21–25
No. of students	11	12	15	9	3

While it is not possible to find the exact mean of a grouped frequency distribution, we can find an estimate of the mean by taking the **mid-interval value** of each class. The mid-interval value in the (1–5) class is found by adding 1 and 5 and dividing by

$$2, \text{ i.e. } \frac{1+5}{2} = 3$$

Similarly, the mid-interval value of the (6–10) class is  $\frac{6+10}{2} = 8$ .

The table given above is reproduced again with the mid-interval values written in smaller print over each class interval.

Mid-interval values	3	8	13	18	23
Marks achieved	1–5	6–10	11–15	16–20	21–25
No. of students	11	12	15	9	3

$$\begin{aligned}\text{mean} &= \frac{\sum fx}{\sum f} \\ &= \frac{(11 \times 3) + (12 \times 8) + (15 \times 13) + (9 \times 18) + (3 \times 23)}{11 + 12 + 15 + 9 + 3} \\ &= \frac{555}{50} = 11.1\end{aligned}$$

### Test Questions 7A

1. Find the mean of each of these arrays of numbers:

(i) 2, 6, 10, 14, 18

(ii) 2, 0, 8, 16, 6, 24

(iii) 6, 0, -5, 4, 8, 5

(iv)  $x, x+3, x+1, x-3, x-1$

11. The ages of children in a youth-club are given in the following table:

Ages (in years)	10-12	12-14	14-16	16-18	18-20
No. of children	12	24	18	12	4

(10-12 means greater than or equal to 10, but less than 12, etc.)

- What is the modal age group?
  - Use the mid-interval value of each class to estimate the mean of the distribution, giving your answer to the nearest half year.
12. The following table shows the weights of parcels posted by a distribution company on a particular day:

Weight (in kg)	0-2	2-4	4-6	6-8	8-10
No. of parcels	6	12	20	48	14

- State the modal class.
  - Calculate the mean of the distribution.
13. The following table gives the number of points scored by a rugby team in 40 matches:

Points scored	0-4	5-9	10-14	15-19	20-24
No. of matches	3	9	15	10	3

Estimate the mean number of points scored per match.

14. Fifty orange boxes were examined and the number of bad oranges in each box was recorded. The results are given in the following table:

No. of bad oranges per box	0-4	5-9	10-14	15-24
No. of boxes	32	8	7	3

Find the mean number of bad oranges per box.

A Cumulative Frequency Graph is a graph plotted from a cumulative frequency table. A cumulative frequency graph is also called an **ogive** or cumulative frequency curve..

### Example 1

Draw a cumulative frequency graph for the frequency table below.

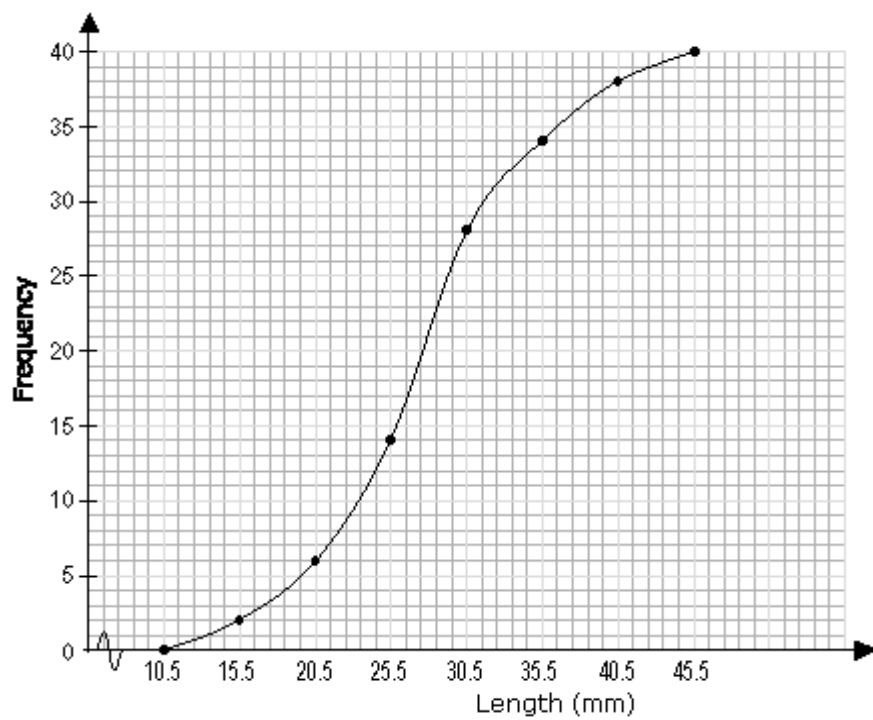
Length (x mm)	Frequency
11 – 15	2
16 – 20	4
21 – 25	8
25 – 30	14
31 – 35	6
36 – 40	4
41 – 45	2

### Solution

We need to add a class with 0 frequency before the first class and then find the upper boundary for each class interval.

Length (x mm)	Frequency	Upper Class Boundary	Length (x mm)	Cumulative Frequency
6 – 10	0	10.5	$x \leq 10.5$	0
11 – 15	2	15.5	$x \leq 15.5$	2
16 – 20	4	20.5	$x \leq 20.5$	6
21 – 25	8	25.5	$x \leq 25.5$	14
25 – 30	14	30.5	$x \leq 30.5$	28
31 – 35	6	35.5	$x \leq 35.5$	34
36 – 40	4	40.5	$x \leq 40.5$	38
41 – 45	2	45.5	$x \leq 45.5$	40

And then plot the cumulative frequency against the upper class boundary of each interval and join the points with a smooth curve.



## Example 2

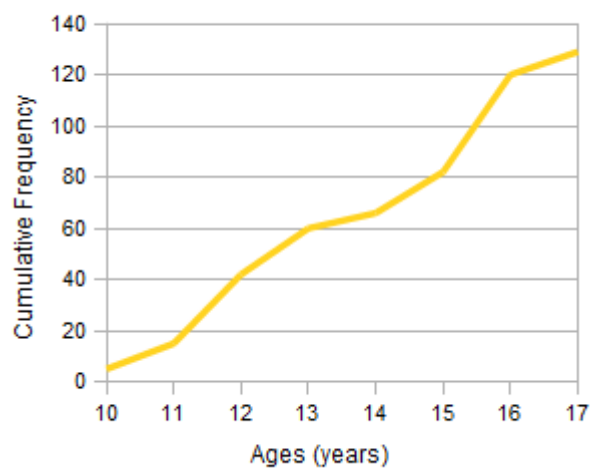
Age (years)	Frequency
10	5
11	10
12	27
13	18
14	6
15	16
16	38
17	9

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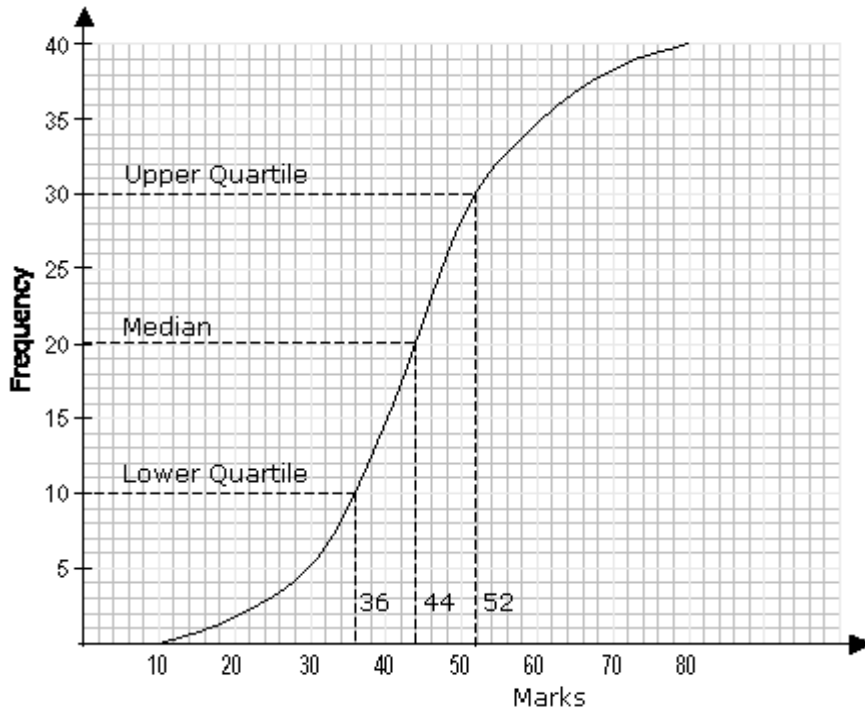
Age (years)	Frequency	Cumulative Frequency
10	5	5
11	10	$5+10 = 15$

12	27	$15+27 = 42$
13	18	$42+18 = 60$
14	6	$60+6 = 66$
15	16	$66+16 = 82$
16	38	$82+38 = 120$
17	9	$120+9 = 129$

Cumulative Frequency Graph (Ogive)



### Exam Results Example - Cumulative Frequency Curve (Ogive)



#### Percentile

A percentile is a certain percentage of a set of data.

#### Median

The median corresponds to the 50th percentile i.e. 50% of the total frequency.

$$\frac{50}{100} \times 40 = \frac{1}{2} \times 40 = 20$$

From the graph, 20 on the vertical axis corresponds to 44 on the horizontal axis. The median mark is 44.

#### Upper Quartile

The upper quartile corresponds to the 75th percentile i.e. 75% of the total frequency.

$$\frac{75}{100} \times 40 = \frac{3}{4} \times 40 = 30$$

From the graph, 30 on the vertical axis corresponds to 52 on the horizontal axis. The upper quartile is 52.

#### Lower Quartile

The lower quartile corresponds to the 25th percentile i.e. 25% of the total frequency.

$$\frac{25}{100} \times 40 = \frac{1}{4} \times 40 = 10$$

From the graph, 10 on the vertical axis corresponds to 36 on the horizontal axis. The lower quartile is 36.

#### Inter Quartile Range

$$\text{Upper} - \text{Lower Quartile} = 52 - 36 = 16$$

3. Teenagers at a disco were asked their ages. The results are shown in the table below:

Age (years)	13	14	15	16	17	18	19
No. of teenagers	4	9	22	19	11	9	6

Copy and complete the cumulative frequency table below.

Age years( $\leq$ )	13	14	15	16	17	18	19
No. of teenagers	4	13					

Draw a cumulative frequency curve and use it to estimate

- the median
  - the interquartile range of the ages of the teenagers.
4. The circumference of each of 150 young trees in a plantation was measured, with the following results:

Circumference (cm)	30–35	35–40	40–45	45–50	50–55	55–60
Frequency	12	25	44	48	18	3

- (a) Copy and complete the following cumulative frequency table:

Circumference ( $\leq$ )	35	40	45	50	55	60
Number of trees	12				147	

- (b) Draw a cumulative frequency graph of the distribution.

From your graph estimate

- the median of the distribution
  - the interquartile range
  - the number of trees with a circumference between 43 cm and 53 cm.
5. The following table gives the percentage increase in a representative sample of 120 grocery items over a four-year period.

Percentage increase	0–20	20–40	40–60	60–80	80–100
No. of items	10	22	62	20	6

Construct a cumulative frequency table and hence draw a cumulative frequency curve.

Use your curve to estimate

- the median percentage increase
- the upper quartile
- the lower quartile
- the interquartile range
- the number of items which increased by more than 55%.

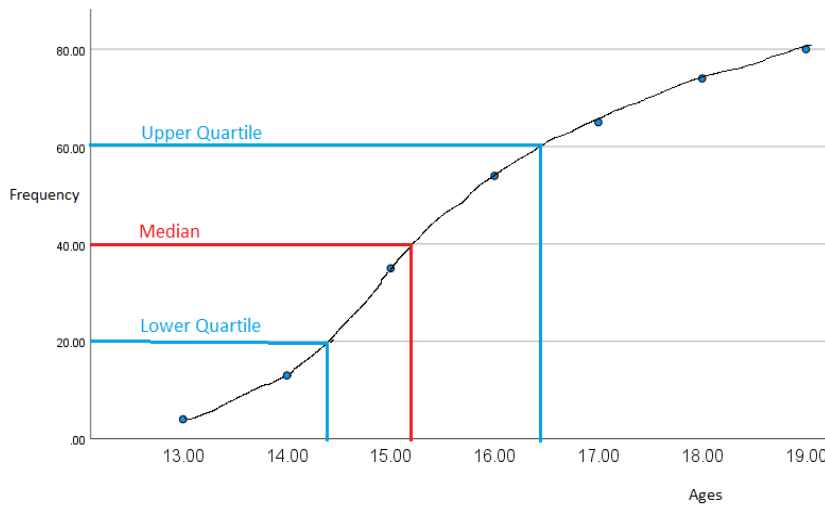
### Ogive – Cumulative Frequency Curve

#### Question 3

Age (x)	13	14	15	16	17	18	19
No. of Teenagers (f)	4	9	22	19	11	9	6

Age	$\leq 13$	$\leq 14$	$\leq 15$	$\leq 16$	$\leq 17$	$\leq 18$	$\leq 19$
No. of Teenagers	4	13	35	54	65	74	80

## Cumulative Frequency Curve (Ogive)



i. **Median** (middle value) = 15.2 years old

ii. **Interquartile Range**

$$\text{Interquartile Range} = \text{Upper Quartile} - \text{Lower Quartile}$$

$$16.4 - 14.4 = 2 \text{ years}$$

**Mean** (arithmetic average)

x	52	126	330	304	187	162	114
f	4	9	22	19	11	9	6

$$\begin{array}{rcl} \text{sum of } fx & = & 1275 \\ \text{sum of } f & = & 80 \end{array}$$

$$\text{Mean} = 15.9375$$

**Mode** (most common value or the value with the biggest frequency)

Mode = 15 years

**Range** (highest x value – lowest x value)

$$19 - 13 = 6$$

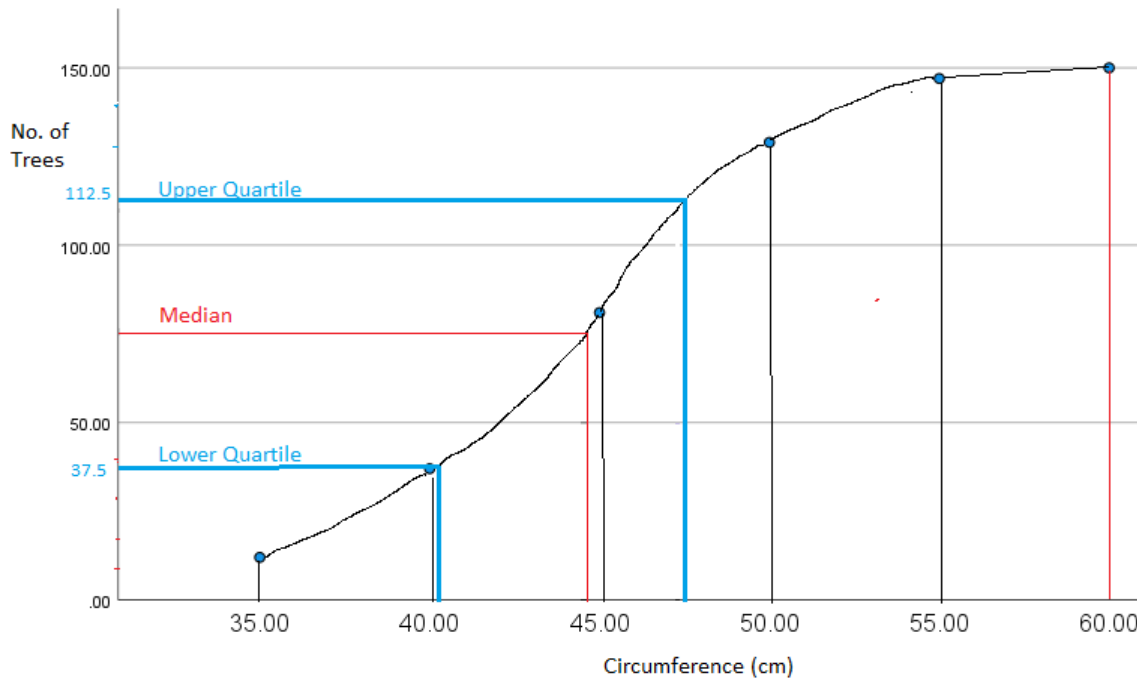
Range = 6 years



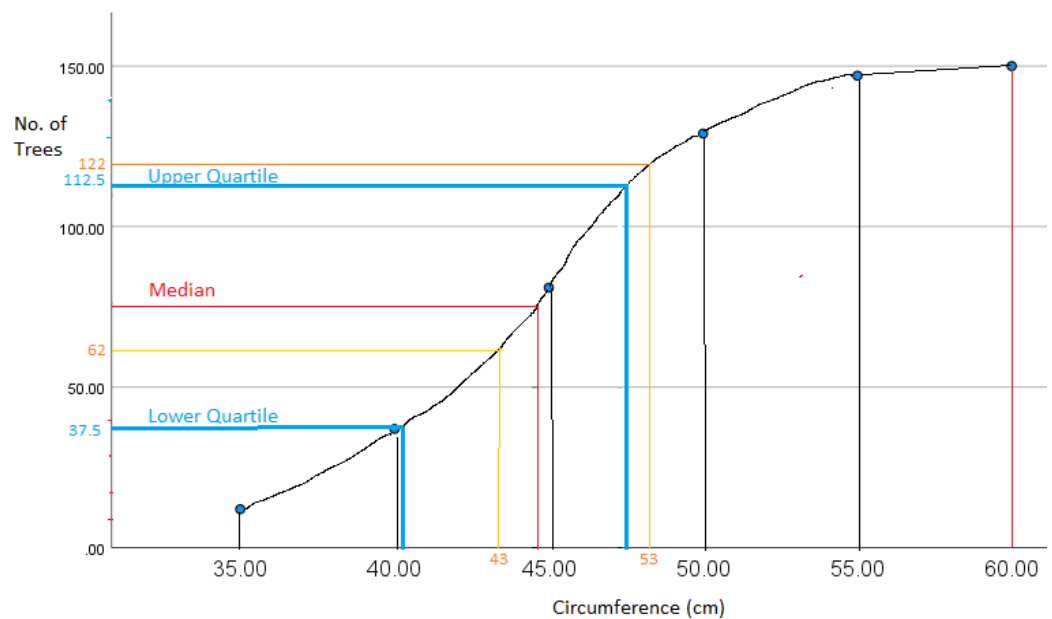
#### Question 4

Circumference (cm)	30_35	35_40	40_45	45_50	50_55	55_60
Frequency	12	25	44	48	18	3

Circumference (cm)	≤35	≤40	≤45	≤50	≤55	≤60
Frequency	12	37	81	129	147	150



- Median** (middle value of 75) = 44 cm
- Interquartile Range**  $\text{Interquartile Range} = \text{Upper Quartile} - \text{Lower Quartile}$   
 $47.5 - 41 = 6.5 \text{ cm}$



- $122 - 62 = 60 \text{ trees}$