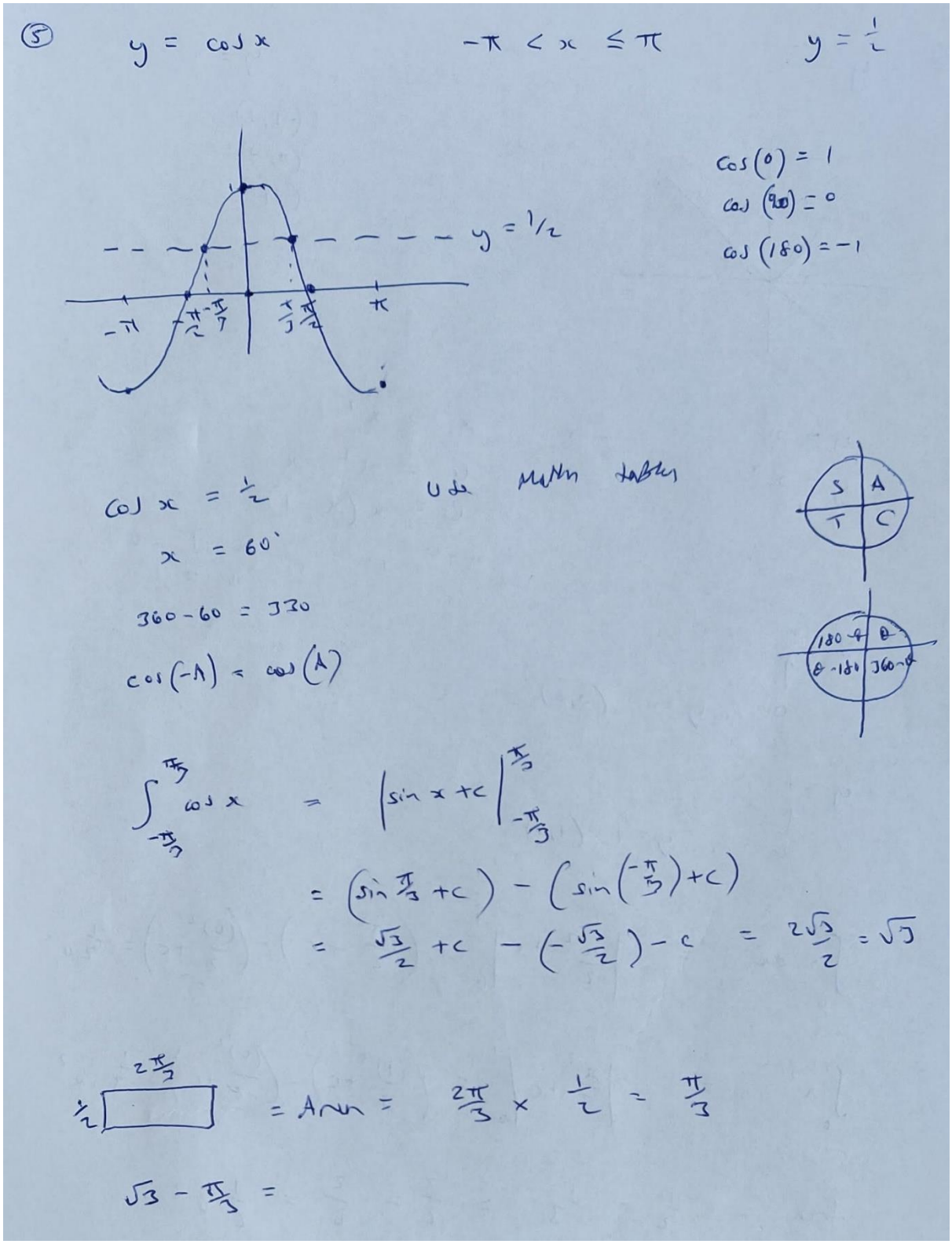
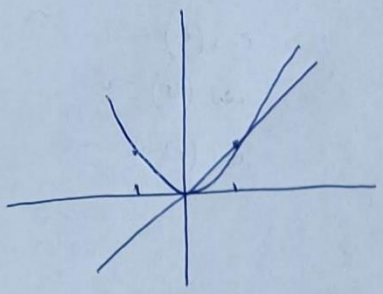
Exercises

5. Calculate the area contained between the curve $y = \cos x$, $-\pi < x < \pi$, and the line $y = \frac{1}{2}$.
6. Find the area contained between the line $y = x$ and the curve $y = x^2$.



⑥ $y = x$ $\qquad$ $y = x^2$

$(-1, 1)$
$(0, 0)$
$(1, 1)$



$y = x$
$y = x^2$ $\longrightarrow$ $x = x^2$

$0 = x^2 - x$

$0 = x(x-1)$

$x = 0$, 1 $\qquad$ $x - 1 = 0$, $x = 1$

$y = x$

$\uparrow$

$0 \therefore y = 0 \quad (0,0)$

$y = 1$

$\uparrow$

$x \therefore y = 1 \quad (1,1)$

$$\int_0^1 x\,dx = \left|\frac{x^2}{2} + c\right|_0^1 = \left(\frac{(1)^2}{2} + c\right) - \left(\frac{(0)^2}{2} + c\right) = \frac{1}{2}\,u^2$$

$$\int_0^1 x^2 = \left|\frac{x^3}{3} + c\right|_0^1 = \left(\frac{(1)^3}{3} + c\right) - \left(\frac{(0)^2}{2} + c\right) = \frac{1}{3}\,u^2$$

$$\text{Area} = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}\,u^2$$

Answers

1. (a) 4 (b) $20\frac{2}{3}$ (c) $10\frac{2}{3}$ (d) $\frac{2}{3}$
(note that the integrals for parts (c) and (d) are negative, but the areas are positive).

2. $\frac{1}{2}$

3. 0. The areas between $x = -1$ and $x = 0$, and between $x = 0$ and $x = 1$, are equal in size. However, one area is above the axis and one is below the axis, so that the integrals have opposite sign.

4. 0. The curve crosses the x-axis at $x = 0$ and $x = 4$. The areas between $x = 0$ and $x = 4$, and between $x = 4$ and $x = 6$, are equal in size. However, one area is above the axis and one is below the axis, so that the integrals have opposite sign.

5. $\sqrt{3} - \pi/3$

6. $\frac{1}{6}$

7. $\frac{1}{3}$