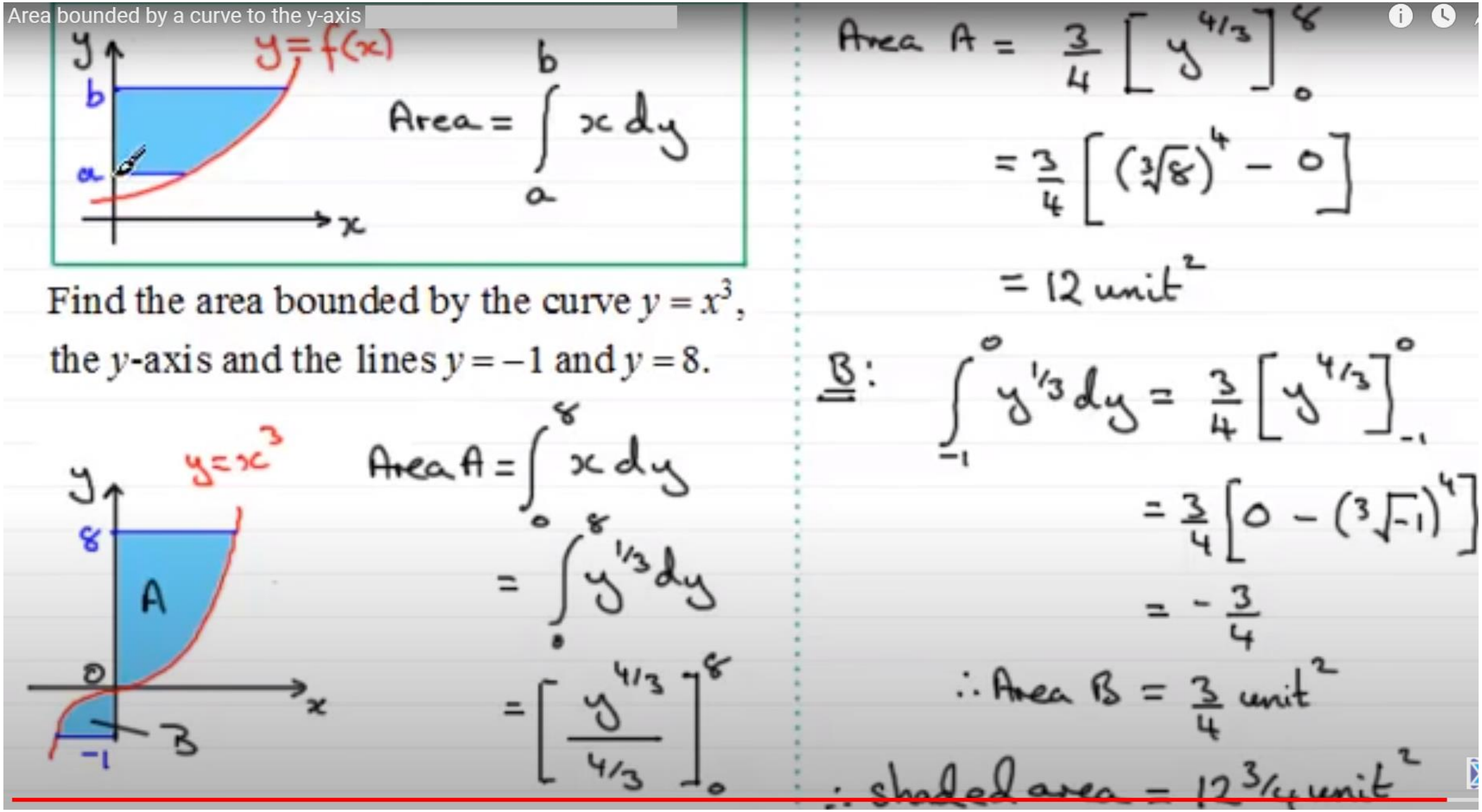
Find the area bounded by the curve $y = x^3$, the y-axis and the lines $y = -1$ and $y = 8$?



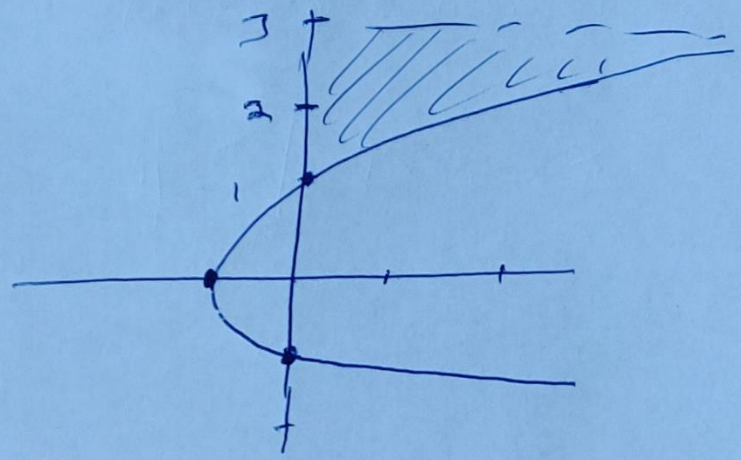
Find the area bounded by the curve $y = \sqrt{x+1}$ the y-axis and the lines $y = 1$ and $y = 3$?

$y = \sqrt{x+1}$

$y^2 = x + 1$

$y^2 - 1 = x$

$\int$



$\boxed{x = 0}$

$y^2 - 1 = 0$

$y^2 = 1$

$y = \sqrt{1}$

$y = \pm 1$

$(0, -1) \quad (0, 1)$

$\boxed{y = 0}$

$(0)^2 - 1 = x$

$-1 = x$

$(-1, 0)$

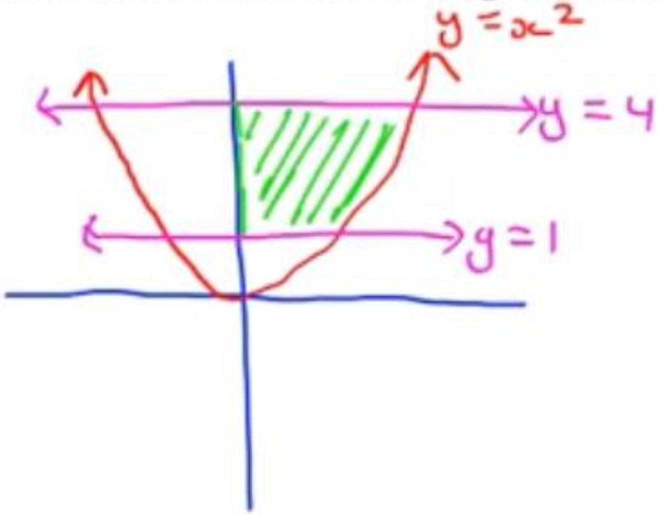
$$\int_1^3 y^2 - 1 = \left| \frac{y^3}{3} - y + c \right|_1^3$$

$$\left(\frac{(3)^3}{3} - (3) + c\right) - \left(\frac{(1)^3}{3} - (1) + c\right)$$

$$9 - 3 + \cancel{c} - \frac{1}{3} + 1 - \cancel{c}$$

$6\,{}^2/_3$ units^2

Calculate the area of the region bounded by the curve $y = x^2$, the y-axis and the lines $y = 1$ and $y = 4$



$$y = x^2$$

$$x = \pm\sqrt{y}$$

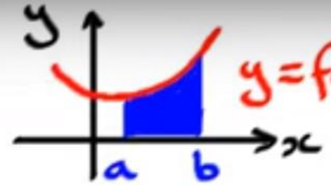
$$x = \pm y^{\frac{1}{2}}$$

$$A = \int_1^4 y^{\frac{1}{2}}\, dy$$

$$= \left[\frac{2}{3} \times y^{\frac{3}{2}}\right]_1^4$$

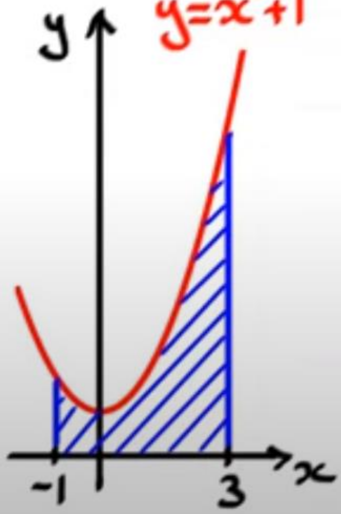
$$= \frac{2}{3} \times 4^{\frac{3}{2}} - \frac{2}{3} \times 1^{\frac{3}{2}}$$

$$= \frac{14}{3} \text{ units}^2$$



$$\text{Area} = \int_a^b f(x)\,dx$$

$$\text{Area} = \int_{-1}^{3} (x^2+1)\,dx$$

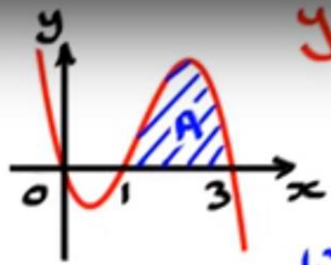


$$= \left[\frac{x^3}{3} + x\right]_{-1}^{3}$$

$$= \left[\frac{(3)^3}{3} + 3\right] - \left[\frac{(-1)^3}{3} + (-1)\right]$$

$$= 12 + \frac{1}{3} + 1$$

$$= \frac{40}{3} \text{ sq. units}$$



$$y = x(1-x)(x-3)$$

$$= x(x-3-x^2+3x)$$

$$= x(4x-x^2-3)$$

$$= 4x^2-x^3-3x$$

When $y=0$, $x=0,1,3$

$$\text{Area } A = \int_1^3 (4x^2-x^3-3x)\,dx$$

$$= \left[\frac{4x^3}{3} - \frac{x^4}{4} - \frac{3x^2}{2}\right]_1^3$$

$$= \left[\frac{4(3)^3}{3} - \frac{(3)^4}{4} - \frac{3(3)^2}{2}\right] - \left[\frac{4(1)^3}{3} - \frac{(1)^4}{4} - \frac{3(1)^2}{2}\right]$$

$$= 36 - \frac{81}{4} - \frac{27}{2} - \frac{4}{3} + \frac{1}{4} + \frac{3}{2}$$

$$= \frac{8}{3} \text{ sq. units}$$

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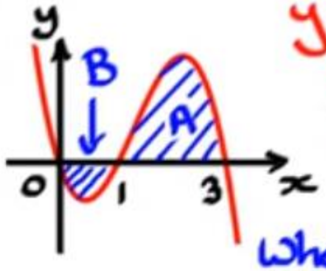

$$y = x(1-x)(x-3)$$
$$= x(x-3-x^2+3x)$$
$$= x(4x-x^2-3)$$
$$= 4x^2-x^3-3x$$

When $y=0$, $x=0,1,3$

$$\text{Area } A = \int_1^3 (4x^2-x^3-3x)\,dx$$
$$= \left[\frac{4x^3}{3} - \frac{x^4}{4} - \frac{3x^2}{2}\right]_1^3$$
$$= \left[\frac{4}{3}(3)^3 - \frac{(3)^4}{4} - \frac{3(3)^2}{2}\right] - \left[\frac{4(1)^3}{3} - \frac{(1)^4}{4} - \frac{3(1)^2}{2}\right]$$
$$= 36 - \frac{81}{4} - \frac{27}{2} - \frac{4}{3} + \frac{1}{4} + \frac{3}{2}$$
$$= \frac{8}{3} \text{ sq.units}$$

$$\int_0^1 (4x^2-x^3-3x)\,dx = \left[\frac{4x^3}{3} - \frac{x^4}{4} - \frac{3x^2}{2}\right]_0^1$$
$$= \frac{4(1)^3}{3} - \frac{(1)^4}{4} - \frac{3(1)^2}{2} - 0$$
$$= -\frac{5}{12}$$

Turns out negative when under the x-axis

$$\therefore \text{Area } B = \frac{5}{12} \text{ sq.units}$$

$$\text{Area } B = -\int_0^1 y\,dx$$

$$\therefore \text{Total Area} = \text{Area } A + B$$
$$= \frac{8}{3} + \frac{5}{12}$$
$$= \frac{37}{12} \text{ sq.units}$$