Area between curve and the Y-axis between two y values

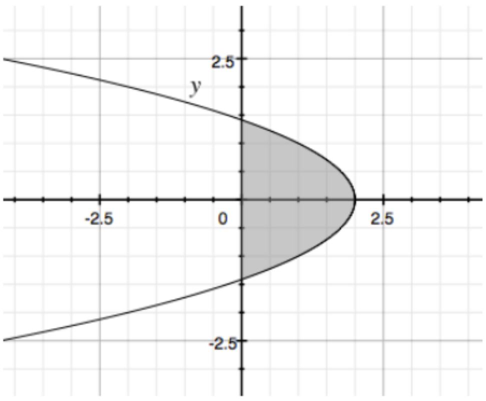
Example 1

Calculate the area trapped between the function $f(y) = -y^2 + 2$ and the y-axis.

Let's first find where the curve $f(y)$ intersects the y-axis. This will be our upper and lower bounds of integration.

$$\begin{aligned} f(y) &= -y^2 + 2 \\ 0 &= -y^2 + 2 \\ 2 &= y^2 \\ y &= \sqrt{(2)}, -\sqrt{(2)} \end{aligned}$$

The following graph represents the area we intend to find:



We can now integrate using the formula from above.

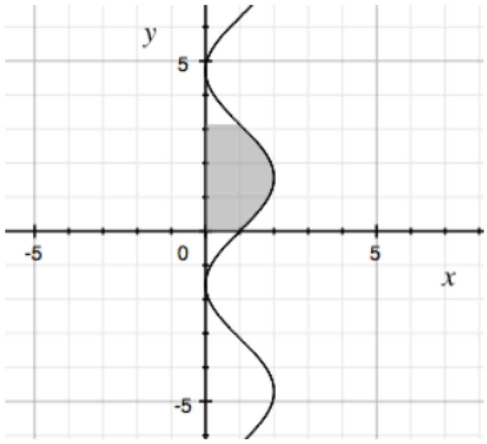
$$\begin{aligned} \textbf{Area} &= \int_{-\sqrt{2}}^{\sqrt{2}} (-y^2 + 2)\, dy \\ \textbf{Area} &= [-\frac{y^3}{3} + 2y]\Big|_{-\sqrt{2}}^{\sqrt{2}} \\ \textbf{Area} &= [-\frac{y^3}{3} + 2y]\Big|_{-\sqrt{2}}^{\sqrt{2}} \\ \textbf{Area} &\approx 3.7712 \end{aligned}$$

Example 2

Calculate the area bounded by the curve $x = \sin y + 1$, $y = 0$, $y = \pi$ and the y-axis.

We note that our lower bound $a = 0$, while our upper bound $b = \pi$, and therefore:

$$\textbf{Area} = \int_0^\pi \sin y + 1\, dy$$
$$\textbf{Area} = [-\cos y + y]_0^\pi$$
$$\textbf{Area} = (-(-1) + \pi) - (-(1) + 0)$$
$$\textbf{Area} = \pi + 2$$



Example 3

Calculate the area bounded by the curve $x = \sqrt{4+y^2}$, $y = -4$, $y = 4$, and the y-axis.

We note that our lower bound $a = -4$ and our upper bound $b = 4$. Therefore:

$$\textbf{Area} = \int_{-4}^{4} \sqrt{4+y^2}\, dy$$

We will now have to use a trigonometric substitution. Let $y = 2\tan\theta$ so that $dy = 2\sec^2\theta\, d\theta$. Making this substitution we get that:

$$\textbf{Area} = \int_{\alpha}^{\beta} \sqrt{4+(2\tan\theta)^2} \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = \int_{\alpha}^{\beta} \sqrt{4+4\tan^2\theta} \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = \int_{\alpha}^{\beta} \sqrt{4(1+\tan^2\theta)} \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = \int_{\alpha}^{\beta} \sqrt{4\sec^2\theta} \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = \int_{\alpha}^{\beta} \sqrt{4\sec^2\theta} \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = \int_{\alpha}^{\beta} 2\sec\theta \cdot 2\sec^2\theta\, d\theta$$
$$\textbf{Area} = 4\int_{\alpha}^{\beta} \sec\theta(1+\tan^2\theta) d\theta$$
$$\textbf{Area} = 4\int_{\alpha}^{\beta} \sec\theta + \sec\theta\tan^2\theta d\theta$$

We will not continue the example further as it is rather tedious, that $A = 23.66$.