Measure Probabilities on a scale from 0 to 1 and assign meanings to points on this scale.

Single Events

When a coin is tossed we know that it is just as likely to come down ‘heads’ as ‘tails’. We say that there is a 50 – 50 chance of showing ‘heads’. So if a coin is tossed ten times, for example, we ‘expect’ to get heads on five of these tosses. Of course this does not always happen. We may get 6 heads and 4 tails or 7 heads and 3 tails.

Now toss a coin 20 times and record the number of harps you get. In general, the greater the number of times you toss a coin the closer you will get to your ‘expected’ result. In an experiment at the beginning of the last century a statistician named Karl Pearson tossed a coin 24,000 times and got 12,012 heads. This, as you can see is very close to the ‘expected’ value of 12,000.

Thus when tossing a coin, we have one chance in two of obtaining a head. Expressing this in a more formal way we say:

The probability of obtaining a head (*H*) is 1/2

We write this as *P* (*H*) = 1/2

Similarly, when a die is thrown, we are equally likely of getting any one of the numbers 1, 2, 3, 4, 5, 6. That is, we have one chance in six of obtaining the number 2, for example, => *P* (*2*) = 1/6

## Example 1

If a card is drawn 200 times from a well shuffled pack of 52. How many of the cards drawn would you expect to be hearts?

1/4 of the cards are hearts. So we expect hearts on 1/4 of 200 occasions i.e. 50 hearts.

*P* (*E*) = number of favourable outcomes in E

number of possible outcomes

**Notes:** **1**. The probability of any event E cannot be less than 0 or greater than 1,

i.e., 0 ≤ *P* (*E*) ≤ 1

**2**. The probability of a certainty is 1.

**3**. The probability of impossibility is 0.

## Example 2

If a card is drawn from a pack of 52, find the probability that it is

(i) a king (ii) a spade (iii) a red card

(i) There are 4 kings in the pack => *P* (king) = 4/52 = 1/13

(ii) There are 13 spades in the pack => *P* (spade) = 13/52 = ¼

(iii) There are 26 red cards => *P* (red card) = 26/52 = ½

Questions

Make out a sample space to show all the possible outcomes when two dice are thrown. Use this to write down the probability that,

1. The sum of the two numbers is 3

# The sum of the two numbers is 9

1. An even number appears on both dice
2. The sum of the two numbers is 10 or more
3. An odd number greater than 1 appears on both dice
4. The difference between the two numbers is 1

A, B, and C are horses equally likely to win a 3-horse race. List all the ways in which the horses can finish, assuming that all the horses finish the race and that there is no dead-heat.

(i) What is the probability that the horses finish in the order A, B and C?

(ii) What is the probability that A wins?

If I throw a die, what is the probability that I get?

(i) 5 (ii) an odd number (iii) a multiple of 3?

A bag contains 5 red beads, 4 black beads and 3 green beads. If one bead is drawn at random from the bag, find the probability that

(i) the bead is red (ii) the bead is green

In a casino a pointer is spun and you win the amount shown in the sector where it comes to rest. Assuming that the pointer is equally likely to come to rest in any sector, what is the probability that you win?

(i) some money (ii) 5 Euros 0 10

(iii) no money (iv) more than 5 Euros

(v) at least 5 Euros? 5 0

20 1

0 5

## The Probability that an Event does not occur

*P* (*E* not occurring) = 1 – *P* (*E*)

## Example

The probability of drawing a spade from a pack of 52 cards is 13/52 = ¼.

Therefore, the probability of not drawing a spade from a deck of 52 is 1 – ¼ = ¾.

## Mutually Exclusive Events – Events A or B

Consider the following two events when drawing a card from a pack of 52:

*A*: drawing an ace

*B*: drawing a king

These events cannot occur together and are said to be **mutually exclusive**. If *A* and *B* are events that cannot occur together, then

*P* (*A* or *B*) = *P* (*A*) + *P*(*B*)

# **Example 1**

A number is selected at random from the numbers 1 – 30 inclusive. Find the probability that it will be either a multiple of 7 or a multiple of 8.

The multiples of 7 are 7, 14, 21, and 28.

The multiples of 8 are 8, 16, and 24.

These events cannot occur together (i.e., they are mutually exclusive).

* + - *P* (multiple of 7 or multiple of 8)
    - = *P* (multiple of 7) + *P* (multiple of 8)
    - = 4/30 + 3/30 = 7/30

# **Note:** If three events *A*, *B* and *C* cannot occur together, then

*P* (*A* or *B* or *C*) = *P (A) + P (B) + P (C)*

**Example 2**

The names Andrew, Barry, Christine and Diana are put in a hat to select two representatives for a competition. Make a list of all the possible pairs of names that could be drawn from the hat. (Use the capital letters A, B, C and D for these names and note that AB is the same as BA.)

Set out a **sample space** giving all the possible outcomes.

*AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB* and *DC*. However, *AB* = *BA* etc. Therefore, we have to remove *BA, CA, CB, DA* and *DC* as they have already been declared. We are then left with a sample space of *AB, AC, AD, BC, BD* and *CD*. Therefore there are 6 possible pairs.

What is the probability of drawing?

(i) Andrew and Barry? (ii) A boy and a girl?

(iii) A pair which includes Diana? (iv) A pair that does not include Barry?

(i) = *P*(*AB*) = number of favourable outcomes in E = 1

number of possible outcomes 6

(ii) = *P (AC)* or *P (AD)* or *P (BC)* or *P (BD*) = 4/6 = 2/3.

(iii) = *P (AD)* or *P (BD)* or *P (CD)* = 3/6 = ½.

(iv) = *P (AC)* or *P (AD)* or *P (CD*) = 3/6 = ½.

## Events that are not Mutually Exclusive

If an event consists of selecting an ace or a heart from a pack of 52 cards, then

*P* (ace) = 4/52 and *P* (heart) = 13/52

However, the number of aces or hearts in a pack of cards is 16 (and not 17, i.e. 13 + 4), because of the 13 hearts and 4 aces, one card is the ace of hearts.

=> *P* (ace or heart) is not equal to *P* (ace) + *P* (heart)

In general, when two events *A* and *B* can occur at the same time, then

*P* (*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*)

Questions

# A card is drawn at random from a pack of 52. What is the probability that the card is

1. A club
2. A king
3. A club or a king
4. A red card
5. A queen
6. A red card or a queen
7. A red card and a queen?

The letters of the word *EXERCISES* are written on 9 cards and placed in a box. If a card is drawn at random, what is the probability that the letter on the card is

* 1. The letter *E*
  2. A vowel
  3. Not the letter *X*
  4. The letter *C* or the letter *E*
  5. The letter *X* and *C*?

**Events A *AND* B – The Multiplication Rule**

If a coin is tossed twice, the result of the first toss has no bearing on the outcome of the second. The two tosses of the coin are said to be **independent** events.

The sample space for tossing a coin twice is {*HH, HT, TH, TT*}

The probability of getting 2 heads is ¼. However, since each trial is independent of the other, we know that the probability of getting a head on the first toss of the coin is ½ and the probability of getting a head on the second toss is also ½.

Multiplying the two probabilities we get ½ \* ½ = ¼.

This is the same answer that was found above by using a sample space. This illustrates the multiplication law of probability, which states that

Probability of *A* and *B* occurring = prob. of *A* occurring \* prob. of *B* occurring

*P* (*A* and *B*) = *P*(*A*) \* *P*(*B*)

## Example

When two dice are thrown, what is the probability of getting 2 sixes?

The probability of getting a 6 with the first die = 1/6

The probability of getting a 6 with the second die = 1/6

=> P(2 sixes) = 1/6 \*1/6 = 1/36.

What if more than two events occur, e.g. what is the probability of getting 5 heads in succession: = ½ \* ½ \* ½ \* ½ \* ½ = 1/32

## Events which are not Independent

For example, if a card is drawn from a pack of 52 and not replaced, the probability of drawing a king is 1/13. Assuming that the first card is a king, the probability of drawing a king on the second card is 3/51, since there are only 3 kings left and 51 cards left.

# Therefore the probability of drawing a king on both cards is 1/13 \* 3/51 =1/221

## Example

A box contains 6 black discs, 4 white discs and 2 green discs. A disc is removed at random and not replaced. A second disc is then removed. Find the probability that

(i) Both discs are black

(ii) The first is white and the second is green

(i) There are 12 discs in the bag.

*P* (1st black) = 6/12 = ½

*P* (2nd black) = 5/11, provided the first was black.

*P* (both black) = ½ \* 5/11 = 5/22

(ii) *P* (1st white) = 4/12 = 1/3

*P* (2nd green) = 2/11, provided the first was white

*P* (1st white and 2nd green) = 1/3 \* 2/11 = 2/33

Questions

A coin is tossed twice. What is the probability of getting

(i) 2 heads (ii) A head on the first and a tail on the second?

A coin is tossed and a die is thrown. What is the probability of getting

(i) A head and a 6 (ii) A tail and an even number

(iii) A head and a multiple of 3?

A bag contains 7 red sweets and 3 yellow sweets. Jane takes a sweet at random and eats it. Barry then also takes a sweet at random. Find the probability that both sweets are

(i) red (ii) yellow

(iii) Find the probability that the first is yellow and the second is red.

A box contains 12 tickets numbered 1 to 12. An event consists of picking at random a ticket from the box and throwing a die.

1. Find the probability of getting 3 on the ticket and 6 on the die.
2. Find the probability of getting the same number on the ticket and on the die.