Rearranging formulae

to change the subject

1. Rearranging the simplest type of equation.

The following formula is used to find the velocity, \mathbf{v} , of an object when you have been given the time, initial velocity and acceleration.

v = u + at

The term **v** is known as the subject of the formula. If we want to find **t** we have to make **t** the **subject of the formula**.

Subtract the initial velocity, **u**, from each side to leave the time, **t**, with the acceleration, **a**:

$$v - u = at$$

To leave **t** on its own, divide both sides by **a**:

$$\frac{v-u}{a} = t$$

Finally rewrite the equation with the subject term on the LHS:

$$t = \frac{v - u}{a}$$

Exercise

Make **d** the subject of the formula:

a = b + cd

2. Rearranging equations with brackets

In the equation shown below, we want to make **y** the subject:

$$\mathbf{x} = \mathbf{k}(\mathbf{y} + \mathbf{z})$$

There is a multiplication on the RHS. To remove ${\bf k}$ we must divide both sides by ${\bf k}:$

$$\frac{x}{k} = y + z$$

Subtract **z** from both sides:

$$\frac{x}{k} - z = y$$

With **y** on the LHS:

$$y = \frac{x}{k} - z$$

Exercise

Rearrange the following to make **u** the subject:

$$s = \frac{1}{2}(u + v)t$$

3. Rearranging equations with fractions

Make **x** the subject of the following equation:

$$\frac{a+4}{x} = 5b$$

Here we have to remove the fraction; we do this by cross-multiplying:

$$a + 4 = 5bx$$

Since we're trying to make **x** the subject we have to get rid of **5b**, so divide both sides by **5b**:

$$\frac{a+4}{5b} = x$$

Rewrite it as:

$$x = \frac{a+4}{5b}$$

We have made **x** the subject.

Exercise

Rearrange the following to make **q** the subject:

$$\frac{m+n}{4} = \frac{q}{2}$$

4. Rearranging equations with quadratics

Make **x** the subject of:

$$\frac{ax^2 + f}{e} = b$$

First we have to get rid of the fraction by cross-multiplying:

 $ax^2 + f = be$

Now subtract **f** from both sides:

 $ax^2 = be - f$

Now we divide both sides by **a** as shown below:

$$x^2 = \frac{be - f}{a}$$

To make **x** the subject we must square root both sides:

$$x = \sqrt{\frac{be - f}{a}}$$

Exercise 4a

Make **u** the subject of:

$$v^2 = u^2 + 2as$$

Exercise 4b

Make **a** the subject of:

$$s = ut + \frac{1}{2}at^2$$

Exam question 1:

When you are **h** feet above sea level, you can see **d** miles to the horizon, where:

$$d = \sqrt{\frac{3h}{2}}$$

Make **h** the subject of the formula.

h =..... (Total 4 marks)

Exam question 2:

Rearrange to make **u** the subject of the formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Give your answer in its simplest form.

(Total 5 marks)