What are factors? Example

- a number or quantity that when multiplied with another produces a given number or expression. In mathematics, a divisor of an integer, also called a factor of, is an integer that may be multiplied by some integer to produce.
- Factors of 12, examples 2 x 6 or 3 x 4

Factorising

- Parts common to a few parts

Factorize the following algebraic expressions:

(a) $6x + 24$

(b) $8x^2 - 4x$

(c) $6xy + 10x^2y$

(d) $m^4 - 3m^2$

(e) $6x^2 + 8x + 12yx$

For the following expressions, factorize the first pair, then the second pair:

(f) $8m^2 - 12m + 10m - 15$

(g) $x^2 + 5x + 2x + 10$

(h) $m^2 - 4m + 3m - 12$

(i) $2t^2 - 4t + t - 2$

(j) $6y^2 - 15y + 4y - 10$

Expand the following, and collect like terms:

(a) $(x + 2)(x - 2)$

(b) $(y + 5)(y - 5)$

(c) $(y - 6)(y + 6)$

(d) $(x + 7)(x - 7)$

(e) $(2x + 1)(2x - 1)$

(f) $(3m + 4)(3m - 4)$

(g) $(3y + 5)(3y - 5)$

(h) $(2t + 7)(2t - 7)$

Factorize the following:

(a) $x^2 - 16$

(b) $y^2 - 49$

(c) $x^2 - 25$

(d) $4x^2 - 25$

(e) $16 - y^2$

(f) $m^2 - 36$

(g) $4m^2 - 49$

(h) $9m^2 - 16$

Quadratics

Expand the following and collect like terms:

(a) $(x + 5)(x + 5)$

(b) $(x + 9)(x + 9)$

(c) $(y - 2)(y - 2)$

(d) $(m - 3)(m - 3)$

(e) $(2m + 5)(2m + 5)$

(f) $(t + 10)(t + 10)$

(g) $(y + 8)^2$

(h) $(t + 6)^2$

Factorize the following:

(a) $y^2 - 6y + 9$

(b) $x^2 - 10x + 25$

(c) $x^2 + 8x + 16$

(d) $x^2 + 20x + 100$

(e) $m^2 + 16m + 64$

(f) $t^2 - 30t + 225$

(g) $m^2 - 12m + 36$

(h) $t^2 + 18t + 81$

Using the -b formula for solving quadratics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factorize the following quadratics using the quadratic formula:

(a) $3x^2 + 2x - 4$

(b) $x^2 + 3x + 1$

(c) $2x^2 + 8x + 3$

(d) $3x^2 + 5x + 1$

(e) $3x^2 + 6x + 2$

(f) $5x^2 + 7x - 2$

(g) $3x^2 + 5x - 4$

(h) $2x^2 + 4x + 1$

(i) $5x^2 + 2x - 2$

(j) $2x^2 + x - 7$

Factorising Examples

- Common parts Example 4x-10 = 2x(2-5)
- Difference of 2 squares
  $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$.

Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Uses of factorising

$$\frac{x^2 - 9}{x - 3} = \frac{(x-3)(x+3)}{(x-3)}$$

$$= \frac{x-3}{x-3} \times (x+3)$$

$$= x + 3$$

$$\frac{x}{x^2 + 4x + 4} + \frac{x}{x+2} = \frac{x}{(x+2)^2} + \frac{x}{x+2}$$

$$= \frac{x}{(x+2)^2} + \frac{x}{x+2} \times \frac{x+2}{x+2}$$

$$= \frac{x}{(x+2)^2} + \frac{x^2 + 2x}{(x+2)^2}$$

$$= \frac{x^2 + x + 2x}{(x+2)^2}$$

$$= \frac{x(x+3)}{(x+2)^2}$$

Factorize and then simplify the following algebraic expressions:

(a) $\frac{x^2+3x}{x+3}$

(b) $\frac{6x^2-8}{2x}$

(c) $\frac{x^2+3x+2}{3x+6}$

(d) $\frac{x^2-7x-18}{x^2-6x-27}$

(e) $\frac{x^2-16}{2x+8}$

(f) $\frac{3x^2-9x}{18x}$