Sum or Difference

If the expression to be differentiated contains more than one term just differentiate, separately, each term in the expression.

Example

(i) If $y = 3x^2 - 5x + 4$, find $\frac{dy}{dx}$.

(ii) If $f(x) = 4x^3 + x^2 - x - 6$, find $f'(x)$.

Solution:

(i) $y = 3x^2 - 5x + 4$

$\frac{dy}{dx} = 6x - 5$

(ii) $f(x) = 4x^3 + x^2 - x - 6$

$f'(x) = 12x^2 + 2x - 1$

Exercise 10.2

Find $\frac{dy}{dx}$ if:

1. $y = x^4$ **2.** $y = x^6$ **3.** $y = 3x^2$ **4.** $y = -5x^4$ **5.** $y = 4x$

6. $y = -3x$ **7.** $y = 8$ **8.** $y = -5$ **9.** $y = \frac{1}{x^3}$ **10.** $y = \frac{1}{x}$

11. $y = x^4 + 2x^3$ **12.** $y = 2x^3 + 5x^2$ **13.** $y = 3x^2 + 4x$

14. $y = 2x^2 - 6x$ **15.** $y = 5x - 2x^2$ **16.** $y = 3x - 1$

17. $y = x^3 + 2x^2 + 5x$ **18.** $y = x - 3x^2 - 4x^3$ **19.** $y = 4 - 5x^2 - 6x^4$

Find $f'(x)$ if:

20. $f(x) = x^2 - x - 6$ **21.** $f(x) = x^3 - 3x^2 + 4$ **22.** $f(x) = 20x - 2x^2$

23. $f(x) = 2x^3 - 8x^2 + 7x - 6$ **24.** $f(x) = x^3 - 2x^2 + 4x - 1$ **25.** $f(x) = 8 + 2x - 3x^2 - x^3$

26. $f(x) = x^3 + \frac{1}{x^3}$ **27.** $3x^2 + \frac{1}{x^2}$ **28.** $f(x) = x^4 + 4 + \frac{1}{x^4}$

Evaluating Derivatives

Often we may be asked to find the value of the derivative for a particular value of the function.

Example

(i) If $y = 2x^3 - 4x + 3$, find the value of $\frac{dy}{dx}$ when $x = 1$.

(ii) If $s = 4t^2 + 10t - 7$, find the value of $\frac{ds}{dt}$ when $t = -2$.

Solution:

(i)
$$y = 2x^3 - 4x + 3$$
$$\frac{dy}{dx} = 6x^2 - 4$$
$$\left.\frac{dy}{dx}\right|_{x=1} = 6(1)^2 - 4$$
$$= 6 - 4 = 2$$

(ii)
$$s = 4t^2 + 10t - 7$$
$$\frac{ds}{dt} = 8t + 10$$
$$\left.\frac{ds}{dt}\right|_{t=-2} = 8(-2) + 10$$
$$= -16 + 10 = -6$$

Exercise 10.3

1. If $y = 3x^2 + 4x + 2$, find the value of $\frac{dy}{dx}$ when $x = 1$.

2. If $y = 2x^3 + 4x^2 + 3x - 5$, find the value of $\frac{dy}{dx}$ when $x = 2$.

3. If $y = 4x^3 - 3x^2 + 5x - 3$, find the value of $\frac{dy}{dx}$ when $x = -1$.

4. If $s = 3t - 2t^2$, find $\frac{ds}{dt}$ when $t = 2$.

5. If $s = t^3 - 2t^2 - t + 1$, find $\frac{ds}{dt}$ when $t = -1$.

6. If $A = 3r^2 - 5r$, find $\frac{dA}{dr}$ when $r = 3$.

7. If $V = 3h - h^2 - 3h^3$, find $\frac{dV}{dh}$ when $h = 2$.

8. If $h = 20t - 5t^2$, find $\frac{dh}{dt}$ when $t = 4$.

9. If $A = \pi r^2$, find $\frac{dA}{dr}$ when $r = 5$, leaving your answer in terms of π.

10. If $V = \frac{4}{3}\pi r^3$, find $\frac{dV}{dr}$ when $r = 3$, leaving your answer in terms of π.

Rule 2: Product Rule

Suppose u and v are functions of x.

If $y = uv$,

then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

In words: First by the derivative of the second + second by the derivative of the first.

Note: The word '**product**' refers to quantities being multiplied.

Example

If $y = (x^2 - 3x + 2)(x^2 - 2)$, find $\dfrac{dy}{dx}$. Hence, evaluate $\dfrac{dy}{dx}$ when $x = -1$.

Solution:

Let $u = x^2 - 3x + 2$ and let $v = x^2 - 2$.

$\dfrac{du}{dx} = 2x - 3$ and $\dfrac{dv}{dx} = 2x$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \quad \text{(product rule)}$$

$$= (x^2 - 3x + 2)(2x) + (x^2 - 2)(2x - 3)$$

$$= 2x^3 - 6x^2 + 4x + 2x^3 - 3x^2 - 4x + 6$$

$$= 4x^3 - 9x^2 + 6$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = 4(-1)^3 - 9(-1)^2 + 6 = -4 - 9 + 6 = -7$$

Exercise 10.4

Use the product rule to find $\dfrac{dy}{dx}$ if:

1. $y = (2x + 4)(3x + 5)$
2. $y = (x^2 + 2)(2x^2 + 7)$
3. $y = (2x + 3)(x^2 + 3x + 4)$
4. $y = (x^2 + 3x)(x^3 - 3x + 4)$
5. $y = (3x^2 - 5x + 3)(3x - 2)$
6. $y = (x + 3)(x - 2)$
7. $y = (x^3 - 2x)(3 - 2x)$
8. $y = (x - 4)(x^3 - 5)$
9. $y = (x^2 + x + 1)(x - 2)$
10. $y = (2x - x^3 - x^4)(x^2 - 3)$
11. $y = (x^3 - x^2 - x)(x^3 + 2x)$
12. $y = (5x^3 - 6x)(x^2 - 3x - 1)$

13. Let $f(x) = (x^2 - 2x)(3x + 2)$. Find $f'(x)$, the derivative of $f(x)$.

14. Let $f(x) = (x^3 - 1)(2x - x^2)$. Find $f'(x)$, the derivative of $f(x)$.

15. If $y = (2x - x^2)(x^2 - x - 1)$, evaluate $\frac{dy}{dx}$ when $x = 0$.

16. If $s = (3t^2 - 4t)(t^2 - 4)$, find $\frac{ds}{dt}$ and evaluate it when $t = 1$.

17. If $x = (2h^2 - 3h + 5)(h - 2)$, evaluate $\frac{dx}{dh}$ when $h = 2$.

18. Find the coefficient of x^3 in the derivative of $(2x^2 - x - 3)(1 - 2x^2)$ with respect to x.

Rule 3: Quotient Rule

Suppose u and v are functions of x.

If $y = \frac{u}{v}$,

then $$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

In words: $$\frac{\text{Bottom by the derivative of the top} - \text{Top by the derivative of the bottom}}{(\text{Bottom})^2}$$

Note: Quotient is another name for a fraction. The quotient rule refers to one quantity divided by another.

Example

If $y = \frac{x^2}{x+2}$ find $\frac{dy}{dx}$ and, hence, find the value of $\frac{dy}{dx}$ when $x = 2$.

Solution:

$$y = \frac{x^2}{x+2}$$

Let $u = x^2$ and $v = x + 2$

$\frac{du}{dx} = 2x$ and $\frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \text{(quotient rule)}$$

$$= \frac{(x+2)(2x) - (x^2)(1)}{(x+2)^2}$$

$$= \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$= \frac{x^2 + 4x}{(x+2)^2}$$

Note: It is usual practice to simplify the top but **not** the bottom.

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{(2)^2 + 4(2)}{(2+2)^2} = \frac{4+8}{(4)^2} = \frac{12}{16} = \frac{3}{4}$$

Exercise 10.5

Find $\frac{dy}{dx}$ if:

1. $y = \frac{3x+2}{x+1}$

2. $y = \frac{2x+1}{x+3}$

3. $y = \frac{x}{x-1}$

4. $y = \frac{5x+2}{x+4}$

5. $y = \frac{1}{x+2}$

6. $y = \frac{1}{x-3}$

7. $y = \frac{3}{x+4}$

8. $y = \frac{x^2}{x-1}$

9. $y = \frac{2x+3}{5x-4}$

10. $y = \frac{2x-3}{x^2+1}$

11. $y = \frac{2x^2}{3x^2-1}$

12. $y = \frac{x^2+2}{4-x^2}$

13. If $y = \frac{3x+2}{x-2}$, evaluate $\frac{dy}{dx}$ at $x = 1$.

14. If $y = \frac{2x^2-1}{x+5}$, evaluate $\frac{dy}{dx}$ at $x = 2$.

15. If $y = \frac{x^2-4x}{5x-1}$, evaluate $\frac{dy}{dx}$ at $x = 0$.

Rule 4: Chain Rule

The chain rule is used when the given function is raised to a power, e.g. $y = (x^2 - 3x + 4)^4$.

To differentiate using the chain rule, do the following in *one* step:

(a) Treat what is inside the bracket as a single variable and differentiate this (multiply by the power and reduce the power by one).

(b) Multiply this result by the derivative of what is inside the bracket.

If $y = (\text{function})^n$,

then $\dfrac{dy}{dx} = n\,(\text{function})^{n-1}$ (derivative of the function).

Example

Find $\dfrac{dy}{dx}$ if: **(i)** $y = (x^2 + 3x)^5$ **(ii)** $y = (2x^2 - 5x + 3)^{20}$.

Solution:

(i) $y = (x^2 + 3x)^5$

$\dfrac{dy}{dx} = 5(x^2 + 3x)^4(2x + 3)$

(ii) $y = (2x^2 - 5x + 3)^{20}$

$\dfrac{dy}{dx} = 20(2x^2 - 5x + 3)^{19}(4x - 5)$

Exercise 10.6

Find $\dfrac{dy}{dx}$ if:

1. $y = (2x + 3)^5$

2. $y = (5x - 1)^4$

3. $y = (x^2 + 3x)^3$

4. $y = (x^2 - 5x - 6)^7$

5. $y = (4 - 5x)^6$

6. $y = (3 - 2x)^5$

7. $y = (1 + x^2)^4$

8. $y = (5 - 2x^2)^7$

9. $y = (4 - 3x - x^2)^8$

10. $y = \left(x^2 + \dfrac{1}{x^2}\right)^5$

11. $y = \left(x^3 - \dfrac{1}{x^3}\right)^4$

12. $y = \left(1 + \dfrac{1}{x}\right)^{10}$

13. If $y = (x^2 - 1)^4$, evaluate $\dfrac{dy}{dx}$ when $x = 1$.

14. If $y = (2x^2 - 3x + 1)^{10}$, find the value of $\dfrac{dy}{dx}$ when $x = 0$.

15. If $y = (h^2 - h + 1)^2$, find the value of $\dfrac{dy}{dh}$ when $h = 1$.

16. If $y = (x^2 + 1)^3$, find the value of $\dfrac{dy}{dx}$ when $x = 1$.

Exercise 10.11

1. If $s = t^3 - 2t^2$, evaluate: (i) $\frac{ds}{dt}$ at $t = 3$ (ii) $\frac{d^2s}{dt^2}$ at $t = 2$.

2. If $h = 4t^3 - 12t + 8$, evaluate: (i) $\frac{dh}{dt}$ at $t = 1$ (ii) $\frac{d^2h}{dt^2}$ at $t = \frac{1}{2}$.

3. A ball bearing rolls along the ground. It starts to move at $t = 0$ seconds. The distance that it has travelled at t seconds is given by $s = t^3 - 6t^2 + 9t$.
   Find:
   (i) $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$, its speed and acceleration, in terms of t
   (ii) the speed of the ball bearing when $t = 4$ seconds
   (iii) the acceleration of the ball bearing when $t = 3$ seconds
   (iv) the times at which the speed is zero
   (v) the time at which the acceleration is zero
   (vi) the time at which the acceleration is 6 m/s^2
   (vii) the time at which the speed is 24 m/s.

4. A particle moves along a straight line such that, after t seconds, the distance s metres from a fixed point o is given by $s(t) = t^3 - 9t^2 + 24t$, $t \geqslant 0$.
   Find:
   (i) $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$, its speed and acceleration, in terms of t
   (ii) the speed of the particle after 6 seconds
   (iii) the times when the speed is zero
   (iv) the acceleration of the particle after 4 seconds
   (v) the time at which the acceleration is zero
   (vi) the time at which the acceleration is 6 m/s^2.

5. The distance, s metres, travelled in t seconds by a train after its brakes are applied is given by $s = 18t - 1.5t^2$.
   Find:
   (i) the distance travelled when $t = 2$ seconds
   (ii) the train's speed, in terms of t
   (iii) the speed of the train when $t = 4$ seconds
   (iv) the time at which the train comes to rest
   (v) the distance travelled by the train after it applied its brakes
   (vi) the constant deceleration of the train.

6. The distance, s metres, travelled by a car in t seconds after the brakes are applied is given by $s = 10t - t^2$. Show that its deceleration is constant. Find:
   (i) the speed of the car when the brakes are applied
   (ii) the distance the car travels before it stops.

7. A ball is thrown vertically up in the air. The height, h metres, reached above the ground t seconds after it was thrown is given by $h = 16t - 2t^2$.

Find:

(i) the height of the ball after 6 seconds

(ii) the speed of the ball in terms of t

(iii) the speed of the ball after 3 seconds

(iv) the height of the ball when its speed is 12 m/s.

(v) After how many seconds does the ball just begin to fall back downwards?
How far above the ground is it at this time?

8. A ball is thrown vertically up in the air. Its height, h metres, above ground level varies with the time, t seconds, such that $h = 1 + 30t - 5t^2$.

Find:

(i) $\frac{dh}{dt}$, its speed

(ii) its speed after $1\frac{1}{2}$ seconds

(iii) its acceleration.

(iv) After how many seconds does the ball just begin to fall back downwards?
How far above the ground is it at this time?

9. The speed, v, in metres per second, of a body after t seconds is given by $v = 2t(6 - t)$.

(i) Find the acceleration at each of the two instants when the speed is 10 m/s.

(ii) Find the speed at the instant when the acceleration is zero.

10. The speed, v, in metres per second, of a body after t seconds is given by $v = 3t(5 - t)$.

(i) Find the acceleration at each of the two instants when the speed is 12 m/s.

(ii) Find the speed at the instant when the acceleration is zero.

11. An automatic valve controls the flow of gas, R cm^3/s, in an experiment. The flow of gas varies with the time, t seconds, as given by the equation $R = 8t - t^2$.

Find:

(i) $\frac{dR}{dt}$, the rate of change of R with respect to t

(ii) the value of $\frac{dR}{dt}$ after 6 seconds

(iii) the time when the rate of flow is a maximum.

(iv) After how many seconds is the rate of flow equal to:

(a) -4 cm^3/s (b) 2 cm^3/s?

12. The volume, V, of a certain gas is given by $V = \frac{20}{p}$, where p is the pressure.

Find:

(i) $\frac{dV}{dp}$, the rate of change of V with respect to p

(ii) the value of $\frac{dV}{dp}$ when $p = 10$.

Exercise 9.7

1. (ii) $\frac{88}{3}$ **2.** 78 **3.** 36 **4.** 32 **5.** 90 **6.** 54

Exercise 10.2

1. $4x^3$ **2.** $6x^5$ **3.** $6x$ **4.** $-20x^3$ **5.** 4

6. -3 **7.** 0 **8.** 0 **9.** $-3x^{-4}$ or $-\dfrac{3}{x^4}$ **10.** $-x^{-2}$ or $-\dfrac{1}{x^2}$

11. $4x^3+6x^2$ **12.** $6x^2+10x$ **13.** $6x+4$ **14.** $4x-6$ **15.** $5-4x$ **16.** 3

17. $3x^2+4x+5$ **18.** $1-6x-12x^2$ **19.** $-10x-24x^3$ **20.** $2x-1$ **21.** $3x^2-6x$

22. $20-4x$ **23.** $6x^2-16x+7$ **24.** $3x^2-4x+4$ **25.** $2-6x-3x^2$

26. $3x^2-3x^{-4}$ or $3x^2-\dfrac{3}{x^4}$ **27.** $6x-2x^{-3}$ or $6x-\dfrac{2}{x^3}$ **28.** $4x^3-4x^{-5}$ or $4x^3-\dfrac{4}{x^5}$

Exercise 10.3

1. 10 **2.** 43 **3.** 23 **4.** -5 **5.** 6 **6.** 13 **7.** -37 **8.** -20 **9.** 10π **10.** 36π

Exercise 10.4

1. $12x+22$ **2.** $8x^3+22x$ **3.** $6x^2+18x+17$ **4.** $5x^4+12x^3-9x^2-10x+12$

5. $27x^2-42x+19$ **6.** $2x+1$ **7.** $-8x^3+9x^2+8x-6$ **8.** $4x^3-12x^2-5$

9. $3x^2-2x-1$ **10.** $-6x^5-5x^4+12x^3+15x^2-6$ **11.** $6x^5-5x^4+4x^3-6x^2-4x$

12. $25x^4-60x^3-33x^2+36x+6$ **13.** $9x^2-8x-4$ **14.** $-5x^4+8x^3+2x-2$

15. -2 **16.** -8 **17.** 7 **18.** -16

Exercise 10.5

1. $\dfrac{1}{(x+1)^2}$ **2.** $\dfrac{5}{(x+3)^2}$ **3.** $\dfrac{-1}{(x-1)^2}$ **4.** $\dfrac{18}{(x+4)^2}$ **5.** $\dfrac{-1}{(x+2)^2}$

6. $\dfrac{-1}{(x-3)^2}$ **7.** $\dfrac{-3}{(x+4)^2}$ **8.** $\dfrac{x^2-2x}{(x-1)^2}$ **9.** $\dfrac{-23}{(5x-4)^2}$ **10.** $\dfrac{-2x^2+6x+2}{(x^2+1)^2}$

11. $\dfrac{-4x}{(3x^2-1)^2}$ **12.** $\dfrac{12x}{(4-x^2)^2}$ **13.** -8 **14.** 1 **15.** 4

Exercise 10.6

1. $5(2x+3)^4(2)$ or $10(2x+3)^4$
2. $4(5x-1)^3(5)$ or $20(5x-1)^3$
3. $3(x^2+3x)^2(2x+3)$
4. $7(x^2-5x-6)^6(2x-5)$
5. $6(4-5x)^5(-5)$ or $-30(4-5x)^5$
6. $5(3-2x)^4(-2)$ or $-10(3-2x)^4$
7. $4(1+x^2)^3(2x)$ or $8x(1+x^2)^3$
8. $7(5-2x^2)^6(-4x)$ or $-28x(5-2x^2)^6$
9. $8(4-3x-x^2)^7(-3-2x)$
10. $5\left(x^2+\frac{1}{x^2}\right)^4(2x-2x^{-3})$ or $5\left(x^2+\frac{1}{x^2}\right)^4\left(2x-\frac{2}{x^3}\right)$
11. $4\left(x^3-\frac{1}{x^3}\right)^3(3x^2+3x^{-4})$ or $4\left(x^3-\frac{1}{x^3}\right)^3\left(3x^2+\frac{3}{x^4}\right)$
12. $10\left(1+\frac{1}{x}\right)^9(-x^{-2})$ or $10\left(1+\frac{1}{x}\right)^9\left(\frac{-1}{x^2}\right)$
13. 0
14. −30
15. 2
16. 24
17. −40
18. 0

Exercise 10.7

1. −1
2. −7
3. 0
4. $3x-y-6=0$
5. $3x-y+5=0$
6. $6x-y+7=0$
7. $x+y-10=0$
8. $3x-y+1=0$
9. $x+y-2=0$; $x-y-3=0$; yes

Exercise 10.8

1. (1, 1)
2. (−2, 13)
3. (4, −1)
4. (3, 0); (−1, −4)
5. (2, −20); (−1, 10)
6. (1, 5)
7. (3, −9); (−2, −4)
8. (0, 0); (−2, 2); $x-y=0$; $x-y+4=0$
9. **(i)** 0 **(iii)** 3 **(iv)** −1, 2
10. −4
11. −3
12. **(i)** −2; 4 **(ii)** $20x-y+28=0$; $20x-y-80=0$

Exercise 10.9

1. (2, −1)
2. (−3, −8)
3. (2, −5)
4. (−3, 16)
5. (−2, 9)
6. (−2, 13)
7. max(−1, 9); min(3, −23)
8. max(1, 3); min(3, −1)
9. max(1, 6); min(2, 5)
10. max(1, 13); min(−3, −19)
11. max(−1, 10); min(−2, 9)
12. max(−2, 20); min(2, −12)
13. **(ii)** $3x^2-18x+24$ **(iii)** max(2, 3); min(4, −1) **(v)** $2<x<4$
14. **(ii)** $a(-1,0)$, $b(0,25)$, $c(1,32)$, $d(5,0)$
15. **(i)** $a=-3$; (2, −2) **(ii)** (0, 2) **(iv)** $-2<k<2$
16. $2px+q$
17. $a=2$, $b=-6$, $c=-3$

Exercise 10.10

1. **(i)** $x < 1$ **(ii)** $x > 1$

2. **(i)** $x < -3$ **(ii)** $x > -3$

Exercise 10.11

1. **(i)** 15 **(ii)** 8

2. **(i)** 0 **(ii)** 12

3. **(i)** $3t^2 - 12t + 9$; $6t - 12$ **(ii)** 9 m/s **(iii)** 6 m/s^2 **(iv)** $t = 1$ or $t = 3$ **(v)** $t = 2$ **(vi)** $t = 3$ **(vii)** $t = 5$

4. **(i)** $3t^2 - 18t + 24$; $6t - 18$ **(ii)** 24 m/s **(iii)** $t = 2$ or $t = 4$ **(iv)** 6 m/s **(v)** $t = 3$ **(vi)** $t = 4$

5. **(i)** 30 m **(ii)** $18 - 3t$ **(iii)** 6 m/s **(iv)** $t = 6$ **(v)** 54 m **(vi)** -3 m/s^2

6. $\frac{d^2s}{dt^2} = -2$ (a constant) **(i)** 10 m/s **(ii)** 25 m

7. **(i)** 24 m **(ii)** $16 - 4t$ **(iii)** 4 m/s **(iv)** 14 m **(v)** $t = 4$; 32 m

8. **(i)** $30 - 10t$ **(ii)** 15 m/s **(iii)** -10 m/s^2 **(iv)** $t = 3$; 46 m

9. **(i)** 8 m/s^2 or -8 m/s^2 **(ii)** 18 m/s

10. **(i)** 9 m/s^2 or -9 m/s^2 **(ii)** 18.75 m/s

11. **(i)** $8 - 2t$ **(ii)** -4 **(iii)** $t = 4$ **(iv)** **(a)** 6 **(b)** 3

12. **(i)** $-20p^{-2}$ or $-\frac{20}{p^2}$ **(ii)** $-\frac{1}{5}$

Exercise 11.1

1. **(i)** €56; €24 **(ii)** 250 g; 200 g

2. **(i)** €120; €160; €200 **(ii)** €1,000; €1,600; €1,400

3. **(i)** 119 g; 34 g; 85 g **(ii)** 72 cm; 54 cm; 36 cm

4. **(i)** €126; €168; €210 **(ii)** 48 cm; 120 cm; 168 cm

5. **(i)** €102; €119; €153 **(ii)** 960 g; 120 g; 480 g

6. 3 : 4 **7.** 1 : 3 **8.** 2 : 1 : 4 **9.** 3 : 2 : 4 **10.** **(i)** €28; €14 **(ii)** 56 g; 224 g

11. **(i)** €60; €120; €30 **(ii)** 39 cm; 156 cm; 390 cm

12. **(i)** 252 g; 168 g; 126 g **(ii)** €240; €320; €360 **13.** A received €15,750; B received €12,250

14. €178,800 **15.** €120 **16.** 100 cm **17.** 162 cm

18. **(i)** €30 **(ii)** €165 **19.** €10,160 **20.** 35 cm **21.** €9,500

22. $k = 5$ **23.** 27.2 km